DETECTABILITY OF A SINGLE SIDEBAND SPREAD SPECTRUM RADIO SIGNAL

Richard Allan Pettus

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THESIS

DETECTABILITY OF A SINGLE SIDEBAND SPREAD SPECTRUM RADIO SIGNAL

Richard Allan Pettus

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Thesis Advisor: Glen Myers

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The analysis demonstrates the susceptibility of the waveform peaks to detection for a variety of bandwidths and noise levels. Results are presented in both graphical and tabular form.

Detectability of a Single Sideband

Spread Spectrum Radio Signal

by

Richard Allan Pettus
Lieutenant, United States Navy
B.S., University of Missouri at Rolla, 1968
M.S., United States Naval Postgraduate School, 1975

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The analysis demonstrates the susceptibility of the waveform peaks to detection for a variety of bandwidths and noise levels. Results are presented in both graphical and tabular form.

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TABLE OF SYMBOLS AND ABBREVIATIONS

$d(t)$	-	Modulating voltage, ideal square wave
$\hat{d}(t)$	-	Modulating voltage, bandwidth limited square wave
m	-	Number of harmonics present in $\hat{d}(t)$
n	-	Envelope of gaussian noise
$p(n)$	-	Pdf of envelope of gaussian noise
$p(R)$	-	Pdf of envelope of SSB signal plus noise
$p(t)$	-	Internally generated function used in correlation receiver
pdf	-	Probability density function
$v_a(t)$	-	Double sideband modulated carrier voltage
$v_c(t)$	-	Single sideband modulated carrier voltage
$x(t)$	-	In phase component of $v_c(t)$
$y(t)$	-	Quadrature component of $v_c(t)$
DSB	-	Double sideband Amplitude Modulation
$E(t)$	-	Instantaneous envelope of $v_c(t)$
G	-	Ratio of peak $v_c(t)$ to rms $v_c(t)$
K	-	Correlation effectiveness of $\hat{d}(t)$
N	-	Average noise power
PD	-	Probability of detection
PFA	-	Probability of false alarm
PG	-	Processing Gain
PN	-	Pseudo-random (pseudo-noise)
R	-	Envelope of SSB signal plus gaussian noise
$R_{pd}(\tau)$	-	Cross-correlation function of $p(t)$ and $d(t)$

ROC - Receiver operating characteristic

S - Average signal power

S_{max} - Peak signal power

SSB - Single sideband Amplitude Modulation

T - period of square wave

V_T - Threshold voltage

σ - RMS noise voltage

$\phi(t)$ - Instantaneous phase angle of $v_c(t)$

ω_c - Radian frequency of carrier

ω_o - Radian frequency of square wave

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I. INTRODUCTION

A. BACKGROUND

Spread spectrum communications include various modulation techniques which produce radio-frequency signals having a bandwidth much wider than the message bandwidth. In most applications the spread spectrum signal is designed to have properties similar to random noise. Such a signal may be difficult to jam and is not easily detected by an uncooperative receiver. Cooperative receivers can, however, use *a priori* information to increase the low received signal to noise ratio to usable levels.

In the most common form of spread spectrum transmission, a high frequency pseudo-random binary sequence (PN sequence) is used as a carrier modulating signal. Since the cooperative receiver correlates the received signal with an identical locally generated sequence, operation is possible with extremely low signal levels at the receiver input [Ref. 1].

Phase reversal modulation has been employed in many digital spread spectrum systems. However, the possibility of modulation removal and subsequent detection of the carrier by an uncooperative receiver has initiated interest in other types of carrier modulation. Single Sideband (SSB) modulation is a type that is of current interest. SSB modulation causes frequency modulation as well as amplitude modulation of the carrier. This composite modulation would seemingly be more

difficult for uncooperative receivers to detect when the ratio of received signal power to noise power is low.

A consideration, then, is the nature of a SSB signal with a bi-level modulating voltage. It is true, as shown in Section II, but perhaps not generally well known, that SSB modulation of a sinusoidal carrier by an ideal square wave results in a sequence of impulse functions. The resulting infinite ratio of peak to average power makes this SSB signal easily detectable in the ideal case. Of interest in this study is the detectability of a non-ideal (realizable) carrier SSB modulated by a bi-level voltage. The dependence on system bandwidth and signal to noise ratio (SNR) of the received signal are considered.

B. SUMMARY OF THE REPORT

The initial indication of infinite peaks of power in the infinite bandwidth SSB waveform leads to the question of detectability of such a signal by an uncooperative receiver. The analysis of the SSB waveform uses a square wave as a model for the PN sequence used in spread spectrum applications. The validity of this model is demonstrated by photographs showing the nearly identical waveforms obtained using a square wave and a PN sequence in a laboratory implementation.

The goal of the analysis is to obtain a realistic description of the SSB waveform and a measure of its detectability under practical conditions. The two most significant limitations are the bandwidth constraints imposed by any practical system and the effects of additive gaussian noise.

In order to examine the shape of the SSB waveform, an analytical expression for the envelope of a square wave SSB signal is derived, evaluated, and plotted for a range of bandwidths of the modulating voltage. The resulting graphs show distinct peaks in the envelope which exhibit increasing definition and magnitude as the bandwidth is increased. For example, if three harmonics of the modulating square wave are transmitted, the ratio of peak to rms signal levels is 1.792, while if fifteen harmonics are transmitted, the corresponding ratio is 2.614.

Bandwidth constraints at the transmitter also affect the ability of a cooperative receiver to recover the data. The results of this study show that larger bandwidths permit greater enhancement of SNR by a cooperative receiver. However, the increase in the ratio of peak signal power to noise power is even greater. For example, the effectiveness of receiver processing increases from 90 per cent of the ideal case to 97 per cent of ideal when the bandwidth is increased from three to fifteen harmonics. But for the same increase in bandwidth, the ratio of peak signal power to noise power nearly doubles. Therefore, increasing the bandwidth improves the effectiveness of a cooperative receiver, but it also increases the magnitude of the peak signal power and the subsequent detectability by a uncooperative receiver. In specific design applications, a trade-off decision must be made based on evaluation of both detectability and receiver effectiveness.

To make intelligent decisions in such a situation, a quantitative measure of detectability is needed. Mathematical definitions of probability of detection (PD) and probability of false alarm (PFA) are developed. By performing a statistical analysis of the SSB waveform in the presence of additive gaussian noise, numerical values for probability of detection and probability of false alarm can be calculated. The results are presented as a family of curves with bandwidth and SNR as parameters.

A comparison of SSB to double sideband (DSB) signals shows that the presence of peaks in the SSB envelope is statistically significant and can be used to differentiate a SSB signal from gaussian noise as well as from a DSB signal of comparable power. For instance, when seven harmonics are transmitted at a SNR of 10 db, comparable probabilities of detection are found to be 0.028 for SSB and 0.00001 for DSB. The data showing the differences in PD for various values of PFA between SSB and DSB is presented for use in design or evaluation of systems when trade-off decisions are made.

II. ANALYSIS AND RESULTS

A. ANALYSIS OF SSB SIGNAL

The initial area of interest is the general shape of the modulated carrier. Such factors as the relative height of peaks, rms values of the signal, and the effect of bandwidth limiting are all of importance when considering the operation of a bandwidth limited radio link.

For an ideal square wave, the modulated carrier is a sequence of impulse functions. To show this, consider the expression for a square wave SSB modulated carrier $v_c(t)$ developed in Appendix A.

$$v_c(t) = \frac{2}{\pi} \Big[\cos (\omega_c + \omega_o) t - \frac{1}{3} \cos (\omega_c + 3\omega_o) t + \frac{1}{5} \cos (\omega_c + 5\omega_o) t \ldots \Big] \quad (1)$$

The carrier frequency is $f_c = \frac{\omega_c}{2\pi}$ and the period of the square wave is $T = \frac{2\pi}{\omega_o}$. The square wave switches levels at times

$$t_k = \frac{T}{4}, \ \frac{3T}{4}, \ \frac{5T}{4}, \ \ldots \ \frac{(2k + 1)T}{4}$$

or when

$$\omega_o t_k = \frac{\pi}{2}, \ \frac{3\pi}{2}, \ \frac{5\pi}{2}, \ \ldots \ \frac{(2k + 1)\pi}{2}$$

Then, for example, equation (1) can be written when k = 0,

$$v_c(t_o) = \frac{2}{\pi}\left[\cos(\omega_c t_o + \frac{\pi}{2}) - \frac{1}{3}\cos(\omega_c t_o + \frac{\pi}{2} + \pi)\right.$$

$$+ \frac{1}{5}\cos(\omega_c t_o + \frac{\pi}{2} + 2\pi)$$

$$\left. - \frac{1}{7}\cos(\omega_c t_o + \frac{\pi}{2} + 3\pi) + \ldots\right]$$

or

$$v_c(t_o) = \frac{2}{\pi}\left[\sin\omega_c t_o + \frac{1}{3}\sin\omega_c t_o + \frac{1}{5}\sin\omega_c t_o\right.$$

$$\left. + \frac{1}{7}\sin\omega_c t_o + \ldots\right]$$

$$= \frac{2}{\pi}\sin\omega_c t_o\left[1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9}\ldots\right] \tag{2}$$

In general,

$$v_c(t_k) = \frac{2}{\pi}(-1)^k \sin\omega_c t_k\left(1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \ldots\right)$$

From Ref. 2,

$$\left(1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \ldots\right) = \infty$$

This development indicates the presence of theoretically infinite peaks twice each cycle in a square wave modulated SSB signal. A PN sequence consists of binary ones and zeroes similar in nature to a square wave, and the same phenomenon of peaks at the switching times can be expected to occur in

PN SSB spread spectrum transmissions. Therefore, SSB may be more susceptible to detection because of peak power variations.

As indicated in Figure 1, a SSB signal may be obtained by using a bandpass filter to selectively suppress one of the sidebands of an amplitude modulated double-sideband suppressed-carrier (AM-DSB/SC) signal. Since the bandpass filter is a linear system, the equation for $v_c(t)$ involves the convolution integral. In general, removal of one sideband with the bandpass filter causes the resulting SSB signal to possess phase modulation as well as amplitude modulation.

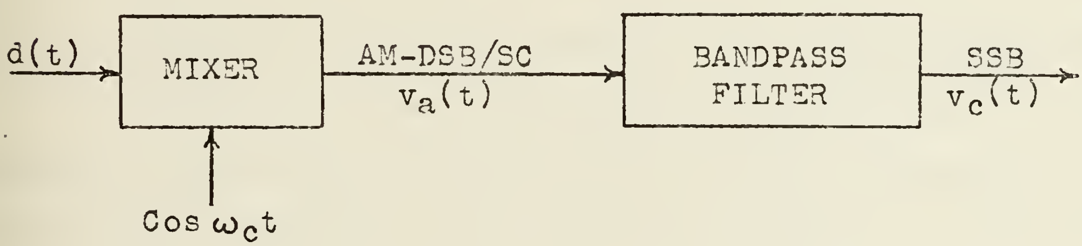

Fig. 1

Block Diagram of a SSB Modulator

1. <u>Square Wave SSB</u>

For the special case where d(t) of Figure 1 is a square wave, it is possible to obtain a series representation of $v_c(t)$ which can be evaluated on a digital computer.

A SSB modulated carrier can be written as

$$v_c(t) = E(t) \quad \mathrm{Cos}\left[\omega_c t + \phi(t)\right]$$

where $E(t)$ is the amplitude modulation (envelope) and $\phi(t)$ is the phase modulation of the carrier. The quantity $E(t)$ is of particular interest since it describes the peaks of $v_c(t)$.

When $d(t)$ is a square wave of period T and $\omega_o = \frac{2\pi}{T}$, it is shown in Appendix A that $E(t)$ is given by

$$E(t) = \left\{ \left[\frac{2}{\pi}(\cos \omega_o t - \frac{1}{3} \cos 3\omega_o t + \ldots) \right]^2 + \left[\frac{2}{\pi} (\sin \omega_o t - \frac{1}{3} \sin 3\omega_o t + \ldots \right]^2 \right\}^{\frac{1}{2}} \tag{5}$$

Equation (5) can be evaluated on a digital computer.

Clearly, one major parameter of equation (5) is the number of terms used in $E(t)$, which, in turn, depends on the width of the bandpass of the filter of Figure 1. The bandwidth of the signal can be specified in the following manner.

If $\hat{d}(t)$ is a bandlimited version of $d(t)$, the corresponding number of harmonics present in $\hat{d}(t)$ can be designated as m. Then, the bandwidth of $v_c(t)$ is given by $\frac{m}{T}$, where, as noted previously, T is the period of the square wave $d(t)$.

Equation (5) can then be evaluated for various values of m. Figures 2-5 are the resulting plots of $E(t)$ for m equal to 3, 7, 11, and 15.

To verify the validity of these simulations as well as to demonstrate the equivalence between square wave SSB and PN SSB signals, the circuit of Figure 1 was constructed.

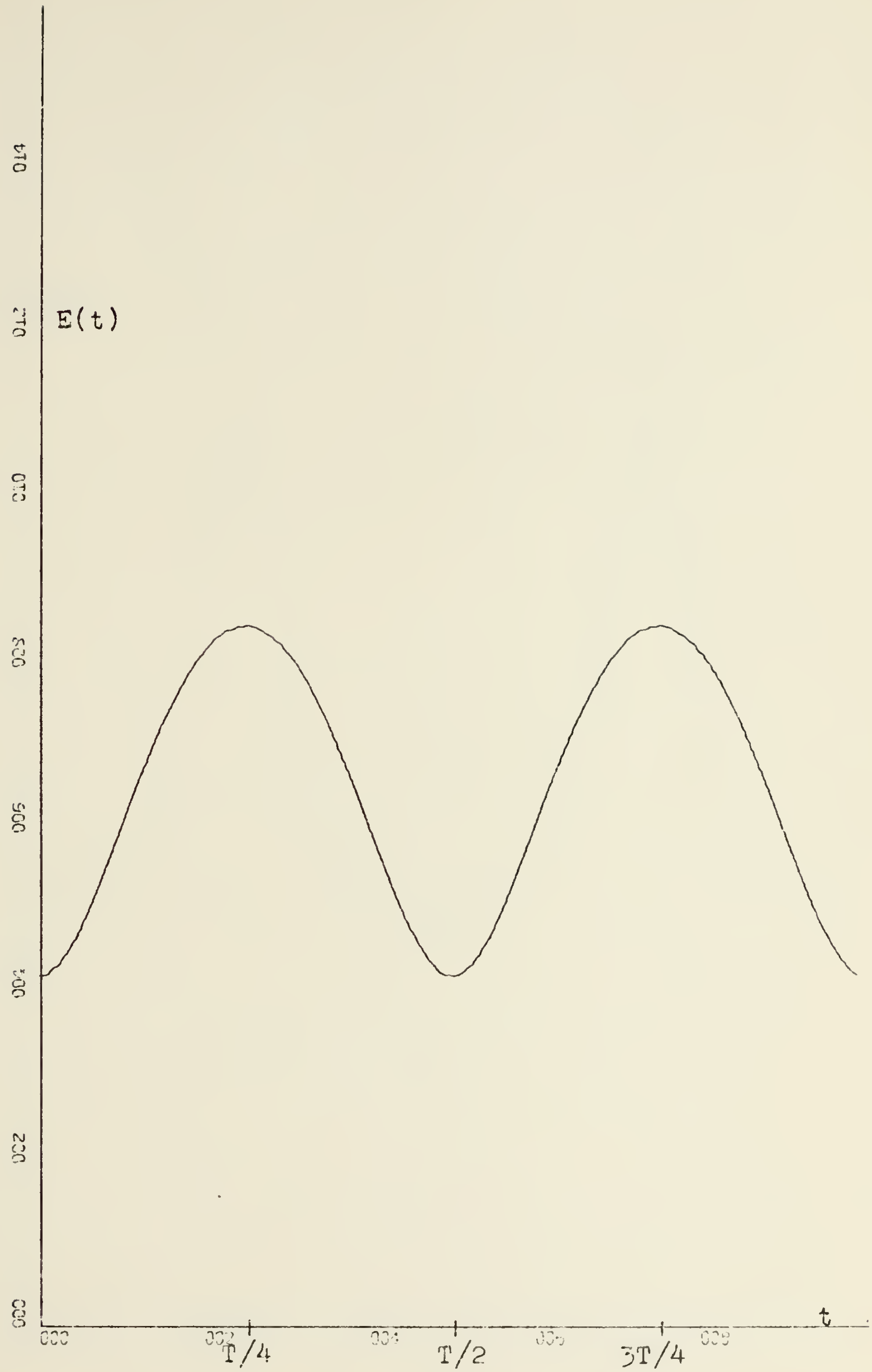

Fig. 2
E(t), m = 3

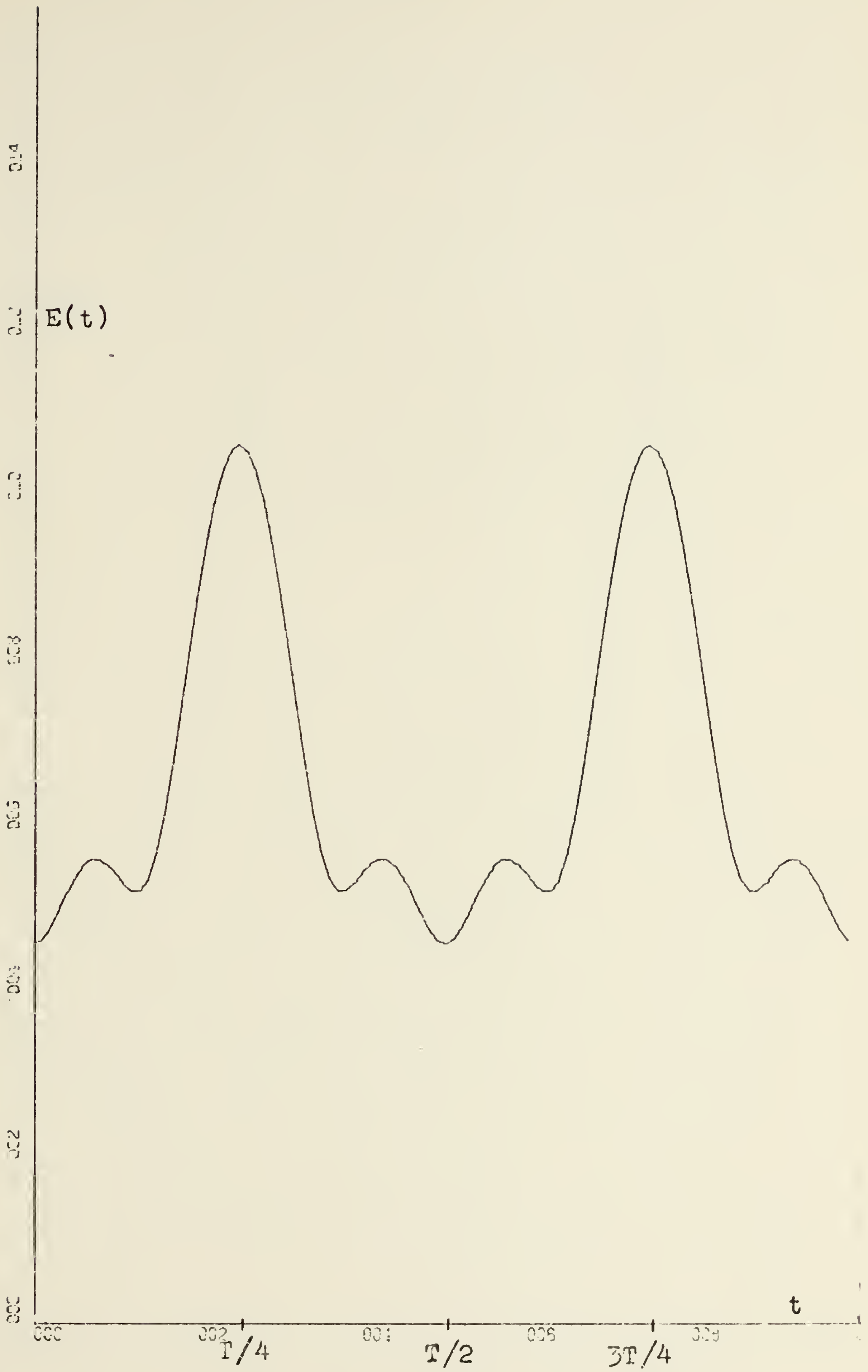

Fig. 3
E(t), m = 7

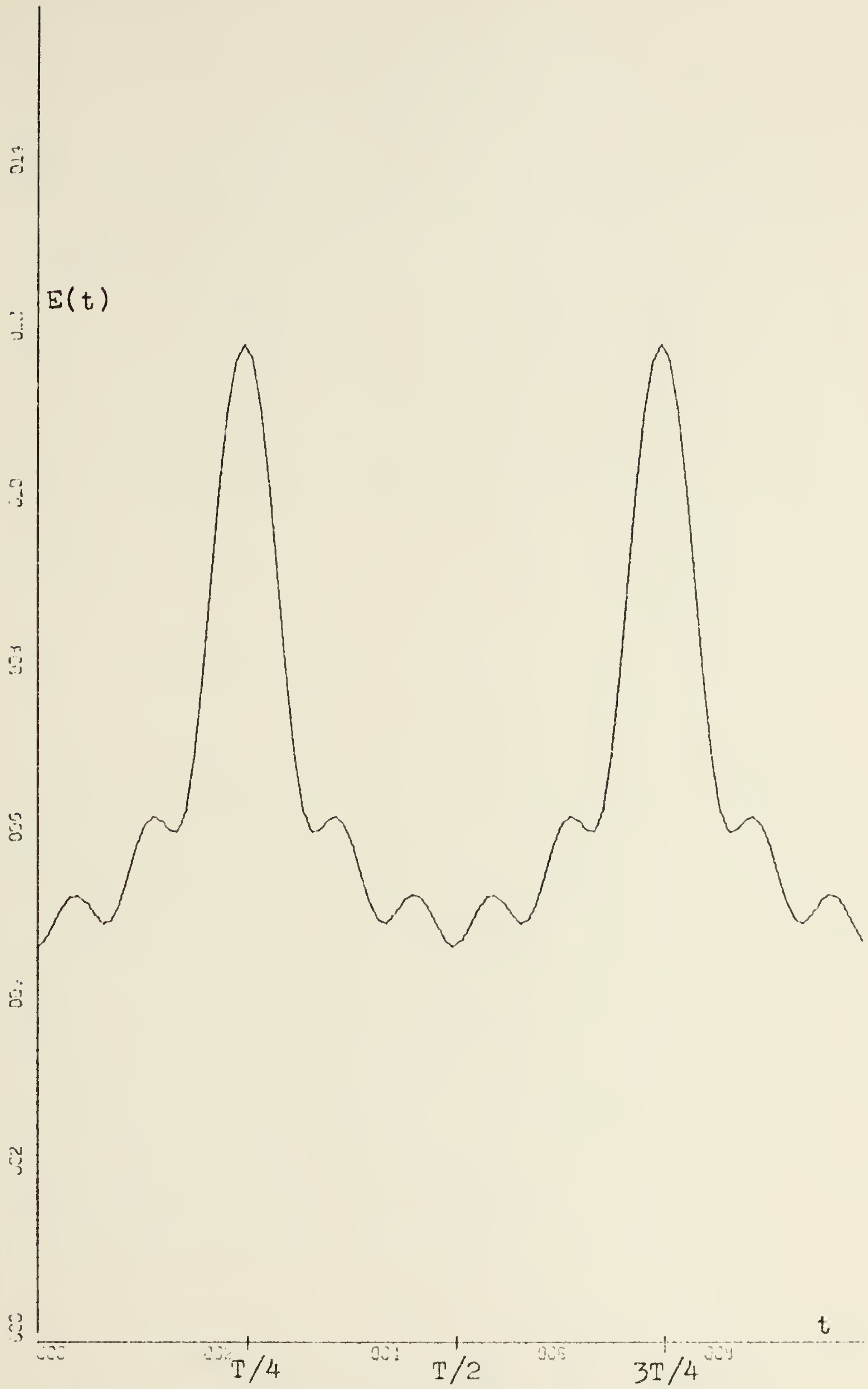

Fig. 4
E(t), m = 11

Fig. 5
E(t), m = 15

An AM-DSB/SC signal is obtained first using a square wave and then using a PN sequence for the modulating voltage d(t). A mechanical filter is then used to select one of the side-bands. Figure 6 is the resulting waveform obtained using a square wave. Figure 7 shows the nearly identical waveform obtained using a PN sequence.

In examing Figures 2-5, one observes that the peaks become more pronounced as a larger number of harmonics are included in d(t). To obtain a quantitative measure of this tendency, the rms and peak values of $v_c(t)$ can be computed and their ratio calculated,

$$G = \frac{v_c(t)_{\text{peak}}}{v_c(t)_{\text{rms}}} \tag{6}$$

Figure 8 shows the dependence of G on m. The increase in G is nearly linear over the range of harmonics simulated. This leads to a preliminary conclusion that the bandwidth of $v_c(t)$ should be minimized. However, other effects are introduced by bandlimiting $v_c(t)$. One important consideration is the effect on a cooperative receiver of limiting the number of harmonics transmitted. This aspect is explored more fully in the next section.

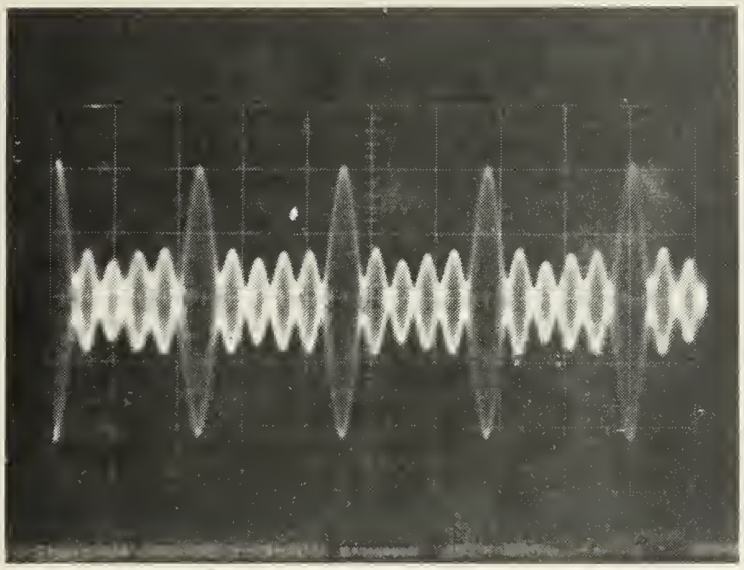

Fig. 6

Square Wave SSB Waveform

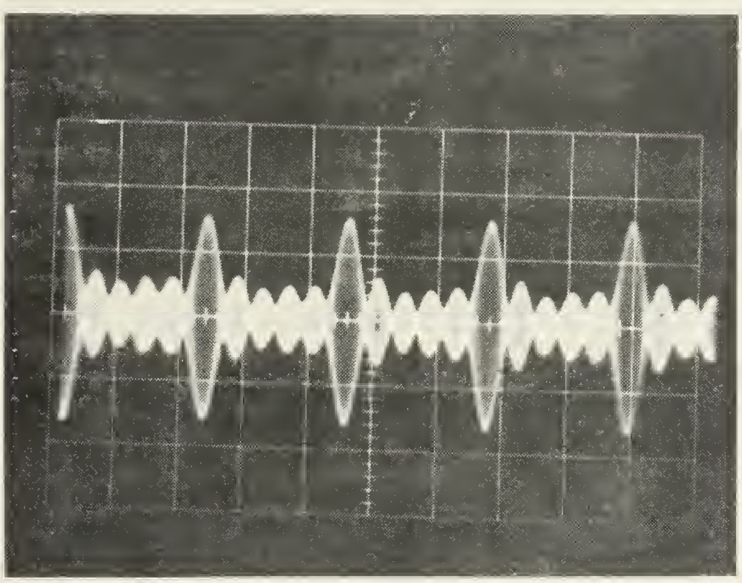

Fig. 7

PN Sequence SSB Waveform

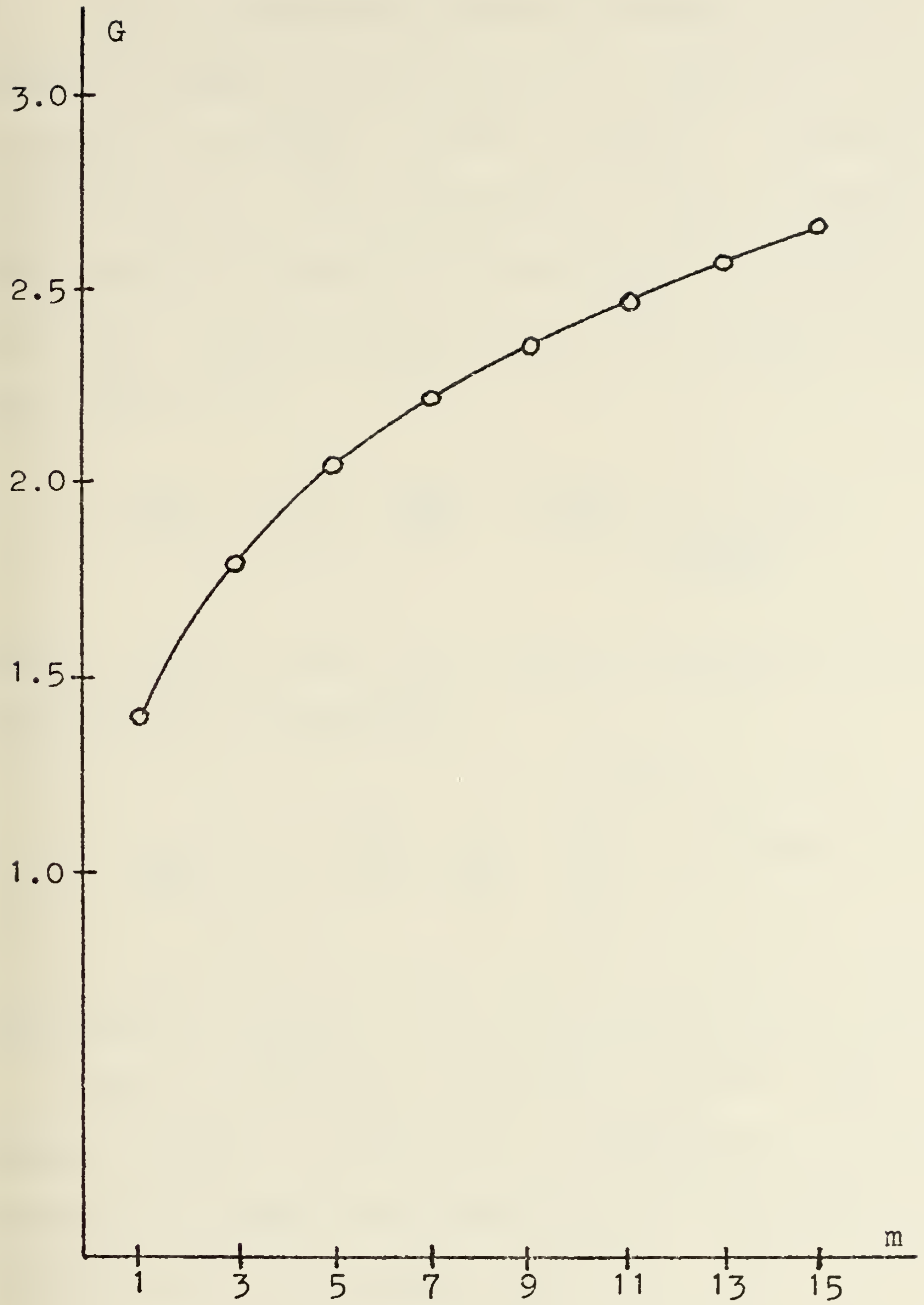

Fig. 8

G vs. m

2. Effect of Bandwidth on Processing Gain

In a correlation receiver (used in a spread spectrum system), *a priori* knowledge of the transmitted signal is used to recover the data. In such a system, a measure of how well the information can be recovered is the processing gain (PG). This quantity defines the improvement of the SNR at the correlator output relative to the SNR at the correlator input [Ref. 1].

$$\left(\frac{S}{N}\right)_{\text{input}} = \frac{1}{PG} \cdot \left(\frac{S}{N}\right)_{\substack{\text{correlator}\\ \text{output}}} \qquad (7)$$

The correlator output is a cross-correlation function $R_{pd}(\tau)$ mathematically defined as

$$R_{pd}(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} p(t)d(t + \tau)\, dt \qquad (8)$$

where d(t) = carrier modulation as previously defined, and

p(t) = internally generated function used in the correlation process. In practice, integration in the receiver is limited to a time T corresponding to the length in seconds of one period of the binary sequence.

Any prior knowledge about d(t) can be used to generate p(t) so that $R_{pd}(\tau)$ is maximized when p(t) is like d(t) and of small value when the input is an interfering signal such as background noise.

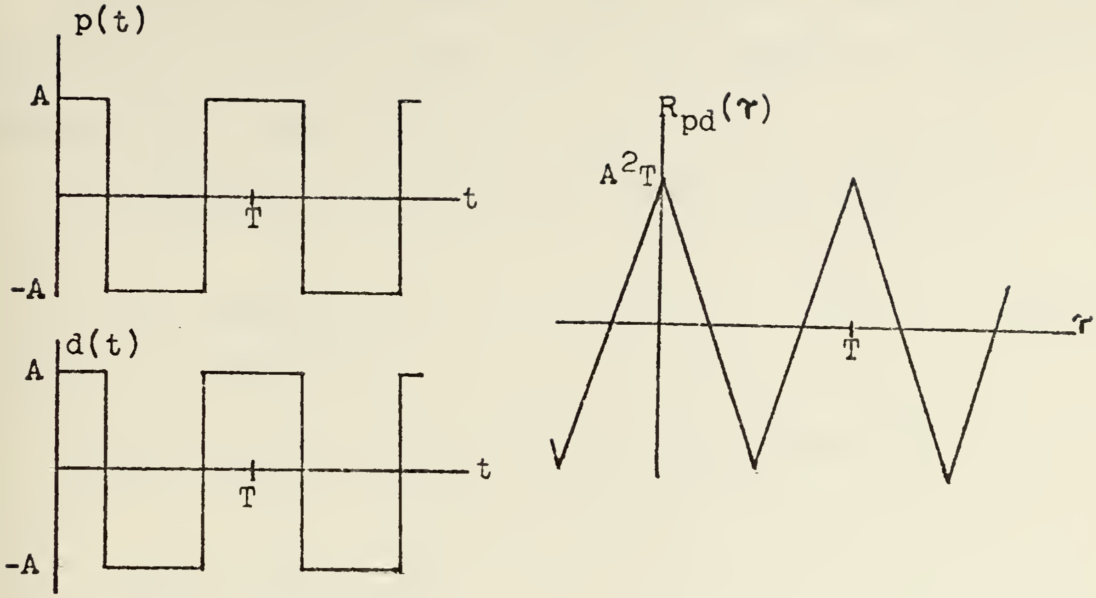

Fig. 9

Cross-Correlation Function of Two Ideal Square Waves

The value of $R_{pd}(0)$ is the total average power in the signal applied to the correlator (assuming p(t) is equal to d(t)).

Now consider the case where d(t) is not an ideal square wave, but a bandwidth limited square wave (band limited $v_c(t)$), designated as $\hat{d}(t)$. Then the maximum peak observed in the ideal case will not be achieved. The ratio of $R_{p\hat{d}}(0)$ to $R_{pd}(0)$ represents the loss in received signal power by correlation with a non-ideal square wave; and, this is a function of the number of harmonics used.

This can be considered to be a loss in processing gain effectiveness by bandlimiting $v_c(t)$. Of interest is the processing gain as a function of m.

To examine this functional dependence, let

$$K = \frac{R_{p\hat{d}}(0)}{R_{pd}(0)} \tag{9}$$

Since $R_{pd}(0)$ is equal to the signal power S,

$$S_{eff} = S_{ideal} \cdot K$$

or,

$$S_{ideal} = \frac{1}{K} S_{eff} \tag{10}$$

The ideal case described by equation (7) can then be modified to give,

$$\left(\frac{S}{N}\right)_{input} = \frac{1}{PG} \cdot \frac{1}{K} \cdot \left(\frac{S_{eff}}{N}\right)_{\substack{\text{correlator} \\ \text{output}}} \tag{11}$$

Equation (11) describes the actual improvement in SNR in a bandwidth limited (non-ideal correlation) case. The parameter K can be interpreted as the effectiveness of the processing gain and is dependent on the bandwidth of $\hat{d}(t)$ transmitted. Equation (8) can be evaluated numerically for values of m from 1 through 15 and the resulting value of K computed. Figure 10 shows the functional relationship of K to m.

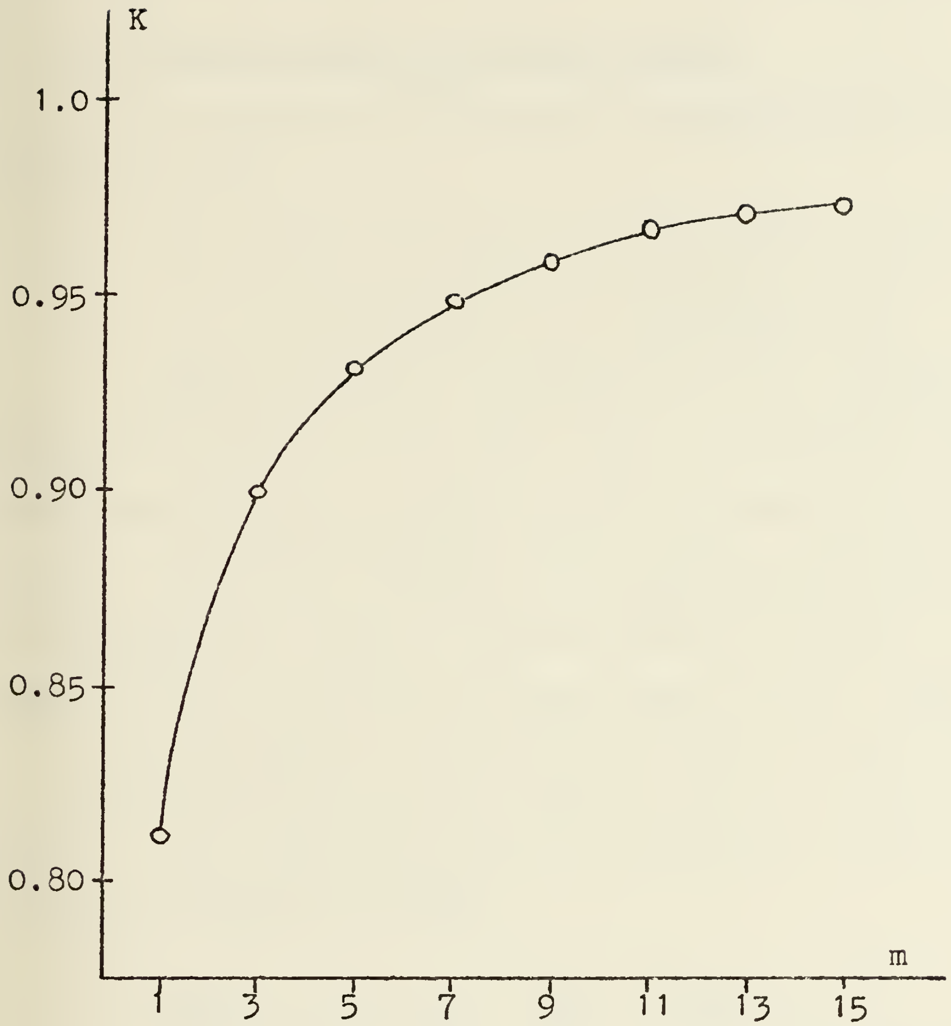

Fig. 10

K vs. m

While there is some dependence of the effectiveness of the processing gain on bandwidth, the result is not significant beyond a small value for m. That is, negligible effects are observed beyond the fifth harmonic of $\hat{d}(t)$.

3. <u>Detectability of PN SSB Signal</u>

Two effects of bandwidth constraints on $v_c(t)$ have been described:

(a) ratio G of peak to rms values of $v_c(t)$

(b) effectiveness K of processing gain

Reduction of the bandwidth of $v_c(t)$ reduces the magnitude of the peaks in the SSB signal, and, hence reduces the detectability of the signal by an uncooperative receiver. However, this also reduces the effective processing gain of the cooperative receiver which implies a greater transmitter power level is required for the same receiver output quality.

If the SNR of the receiver input, called $(S/N)_{RF}$ here, is the same as that at the correlator input, equation (11) can be used to obtain

$$\left(\frac{S}{N}\right)_{RF} = \frac{1}{K} \cdot \frac{1}{PG} \cdot \left(\frac{S}{N}\right)_{\substack{\text{correlator}\\ \text{output}}} \tag{12}$$

Now, designating the peak signal power in the SSB signal as S_{max} and using equation (6) gives,

$$S_{max} = G^2 S \tag{13}$$

Substituting equation (13) into equation (12) gives

$$\left(\frac{S_{max}}{N}\right)_{RF} = \frac{G^2}{K} \cdot \frac{1}{PG} \cdot \left(\frac{S}{N}\right)_{\substack{\text{correlator}\\ \text{output}}} \qquad (14)$$

The quantity S_{max}/N is a measure of the detectability of the SSB signal by an uncooperative receiver. Equation (14) relates this quantity to both G and K (each dependent on bandwidth) in such a manner that the effect on processing gain and peaks in the SSB signal are both taken into account. The dependence of the ratio G^2/K to bandwidth then shows how the detectability of a SSB signal varies with bandwidth of the modulating voltage transmitted. The plot in Figure 11 shows a nearly linear increase in the detectability with increasing m.

These results illustrate the increased detectability of a SSB signal when the envelope peaks are considered. For example, if a SNR of +10 db is required by the receiver after correlation and a PG of +20 db is used, equation (7) gives an RF SNR of -10 db at the receiver input. However, by using equation (14) and Figure 11, the RF peak signal to noise ratio is found to be -6 db for m = 3 and -1.5 db when m = 15.

When SSB signals are being considered, the relation of peak to average signal levels as given by equation (14) is significant, and Figure 11 can be used to determine the numerical correction for a given bandwidth.

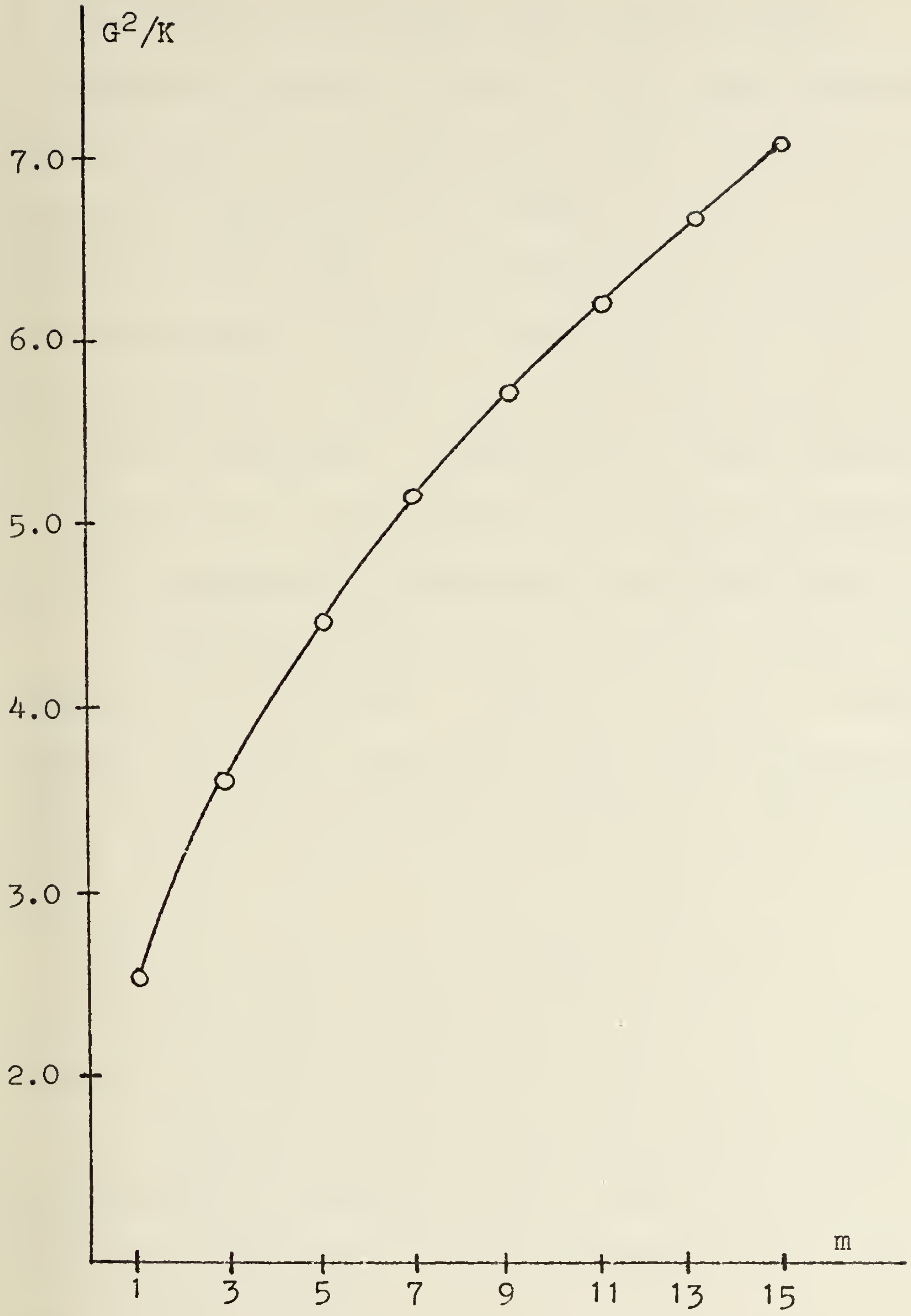

Fig. 11

G^2/K vs. m

B. ANALYSIS OF SSB SIGNAL WITH ADDITIVE NOISE

Preceding sections describe the peaking phenomenon of the SSB signal in deterministic terms. However, in order to include an analysis of the effects of noise, a statistical approach is required. The probability density function (pdf) of the envelope of the SSB signal plus noise provides a useful description and is a tool which can be used as a measure of system performance. Detectability can be measured in terms of probability of detection and probability of false alarm.

1. Probability of Detection and Probability of False Alarm

The background noise is assumed to have a gaussian amplitude distribution. If gaussian noise is applied to an envelope detector, the output amplitude is Rayleigh distributed [Ref. 3]. The pdf of the envelope, n, of a gaussian signal is,

$$p(n) = \frac{n}{\sigma^2} \exp\left(-n^2/2\sigma^2\right), \quad n \geq 0 \tag{15}$$

where σ^2 = average noise power.

Figure 12 is a plot of the Rayleigh distribution of the envelope of gaussian noise. Also included is a typical plot of the pdf p(R) of the envelope R(t) of a SSB signal plus additive noise.

A threshold voltage V_T can be arbitrarily chosen. Voltage above the threshold is considered to be caused by the presence of the SSB signal, and voltage below the threshold is assumed to be from the noise alone.

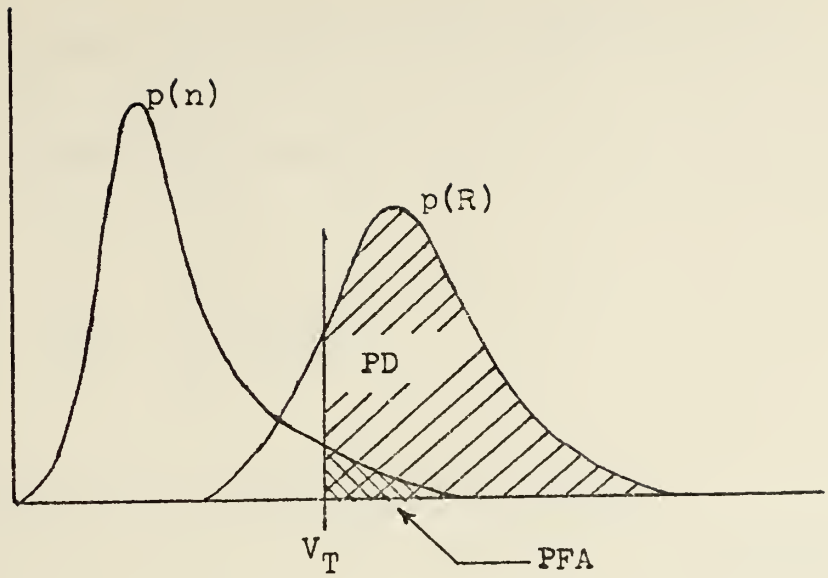

Fig. 12
Graphical Interpretation of PD and PFA

Obviously, such a decision rule will be in error at times. Occasionally, the noise alone will be great enough to be above the threshold. The probability that this will happen is called the probability of false alarm (PFA). Also, at times, the signal may be present, but with amplitude below the threshold. Therefore, the probability that the signal is above the threshold is designated as the probability of detection (PD).

The graphical interpretation of PD and PFA follow from the definition of a pdf. PD is the area under the signal pdf above V_T, while PFA is the corresponding area under the noise pdf.

The equivalent mathematical expressions are

$$PFA = \int_{v_T}^{\infty} p(n)dn$$

$$PD = \int_{v_T}^{\infty} p(R)dR \qquad (16)$$

2. <u>Statistical Analysis of SSB Signal</u>

The first step in the analysis is to transform E(t) into a pdf p(E). Since E(t) was previously determined numerically, a histogram can be constructed by counting the relative frequency of occurence of each amplitude level. The result is an accurate approximation to p(E).

The actual quantity of interest is the pdf of the envelope of the SSB signal plus additive gaussian noise. This can be determined in the following manner [Ref. 4]. For a given value of $E(t) = E_o$, the probability density function of the combined signal plus gaussian noise is given by,

$$p(R/E) = \frac{R}{\sigma^2} \exp\left[-\left(\frac{R^2 + E_o^{\,2}}{2\sigma^2}\right)\right] I_o\left[\frac{R \cdot E_o}{\sigma^2}\right] \qquad (17)$$

where, σ^2 = average noise power

E_o = given value for E(t)

I_o = Bessel Function of zero order and purely imaginary argument

R = envelope of signal plus noise

The next step is to extend this for all possible values of $E(t)$ and determine an overall pdf for $R(t)$, the instantaneous envelope of signal plus noise. The joint pdf $p(R,E)$ is given by

$$p(R,E) = p(E) \cdot p(R/E) \tag{18}$$

The marginal pdf $p(R)$ is then given by,

$$p(R) = \int_{-\infty}^{+\infty} p(R,E)\, dE \tag{19}$$

These expressions can be evaluated numerically on a digital computer for selected values of SNR and bandwidth of the modulating voltage. Figures 13-17 show the pdf of the envelope obtained from a SSB signal plus noise for SNR of +10 db, +3 db, 0 db, -3 db, and -10 db respectively. The corresponding pdf of the envelope of gaussian noise is also shown on each graph. Note the scale change of the ordinate of Figures 16 and 17.

When the SNR is large (+10 db), the presence of peaks in the envelope is shown by the secondary plateau in the pdf for large amplitude values. As the SNR is reduced, the additive noise obscures this feature, but significant separation between the signal pdf and the noise pdf remains. This makes possible a reasonably high probability of detection for a low probability of false alarm. Finally, at a SNR of -10 db, the signal is nearly indistinguishable from the noise.

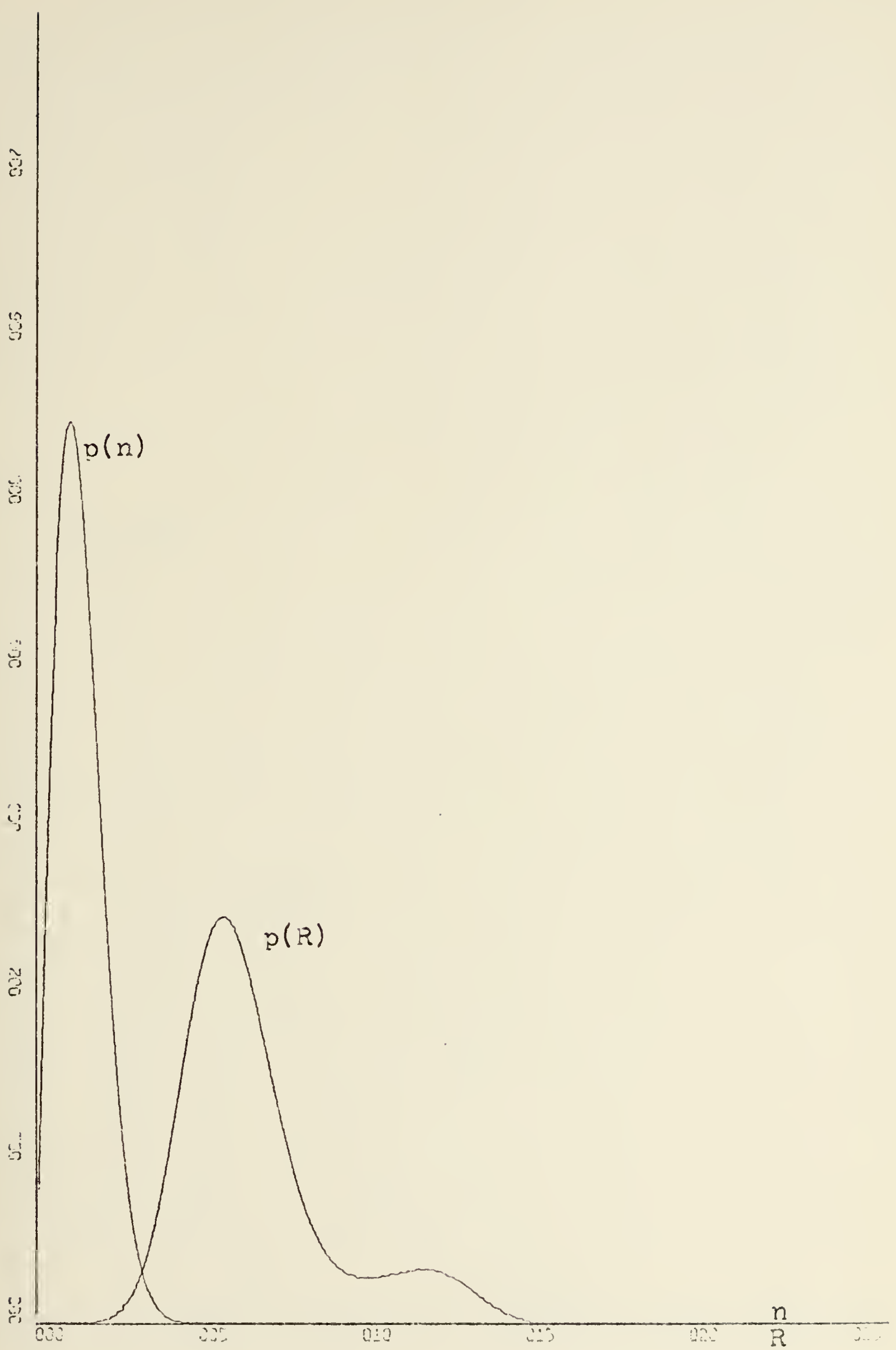

Fig. 13
Envelope PDF p(R), SNR=+10 db

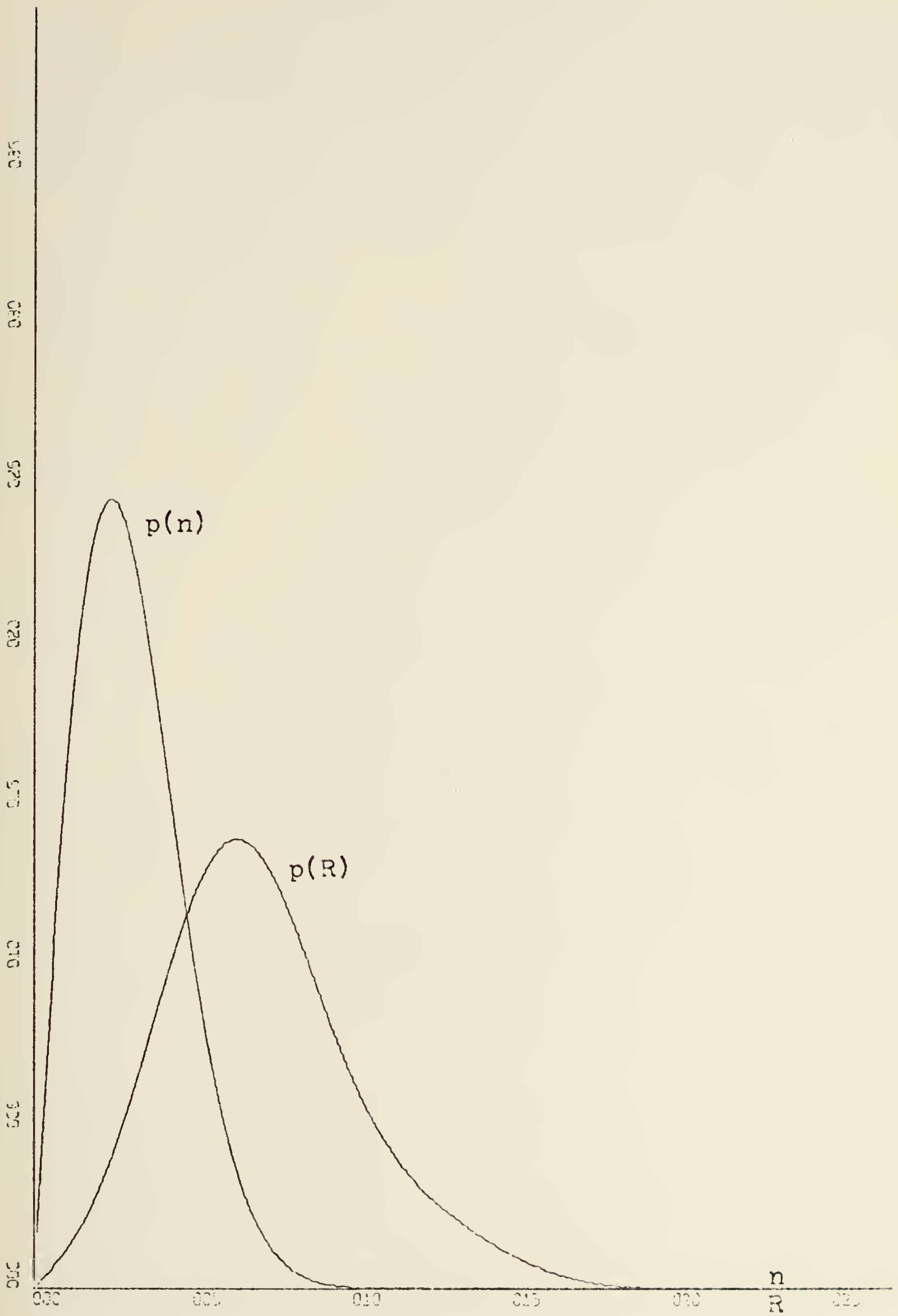

Fig. 14
Envelope PDF p(R), SNR=+3 db

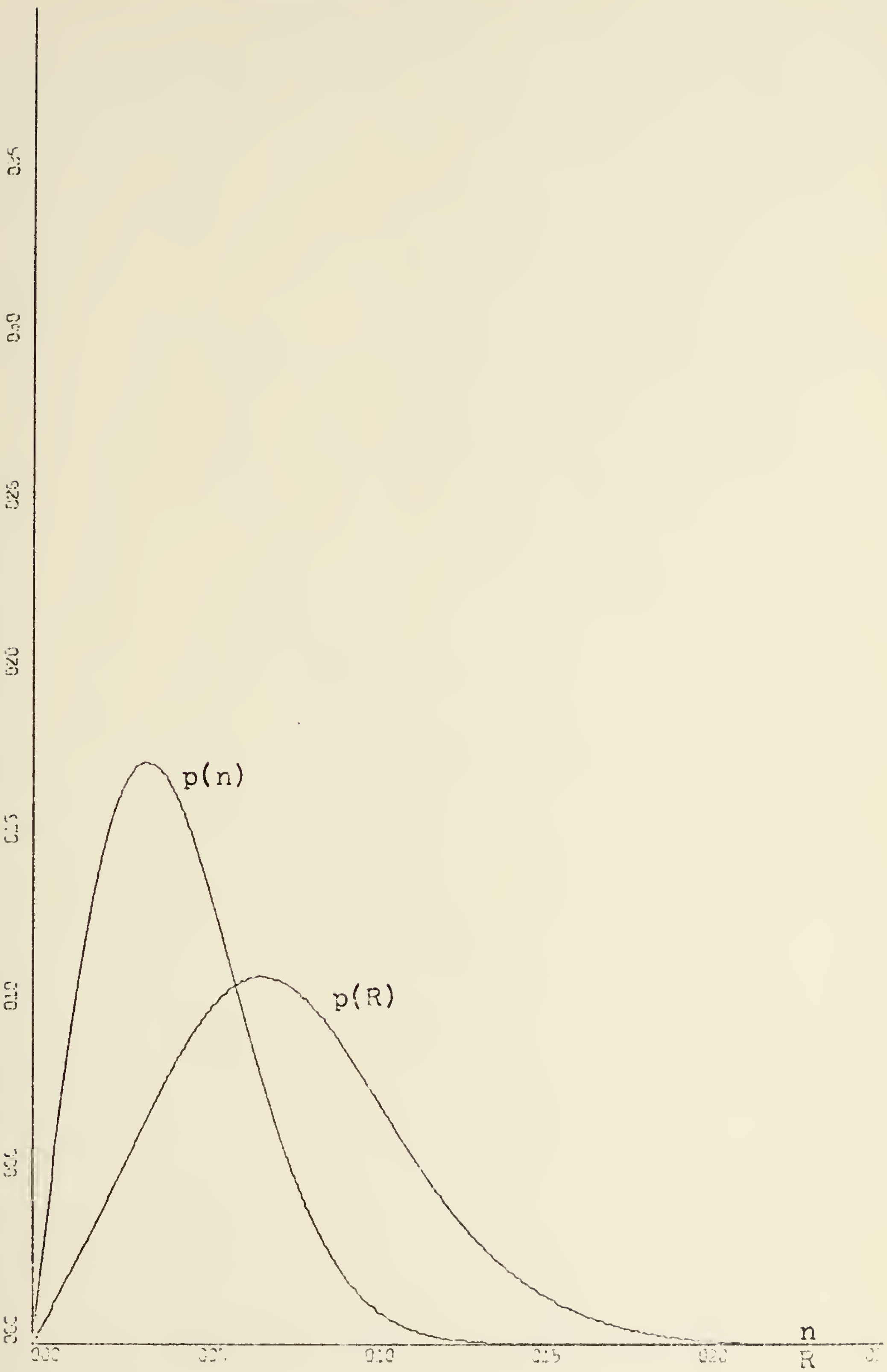

Fig. 15
Envelope PDF p(R), SNR=0 db

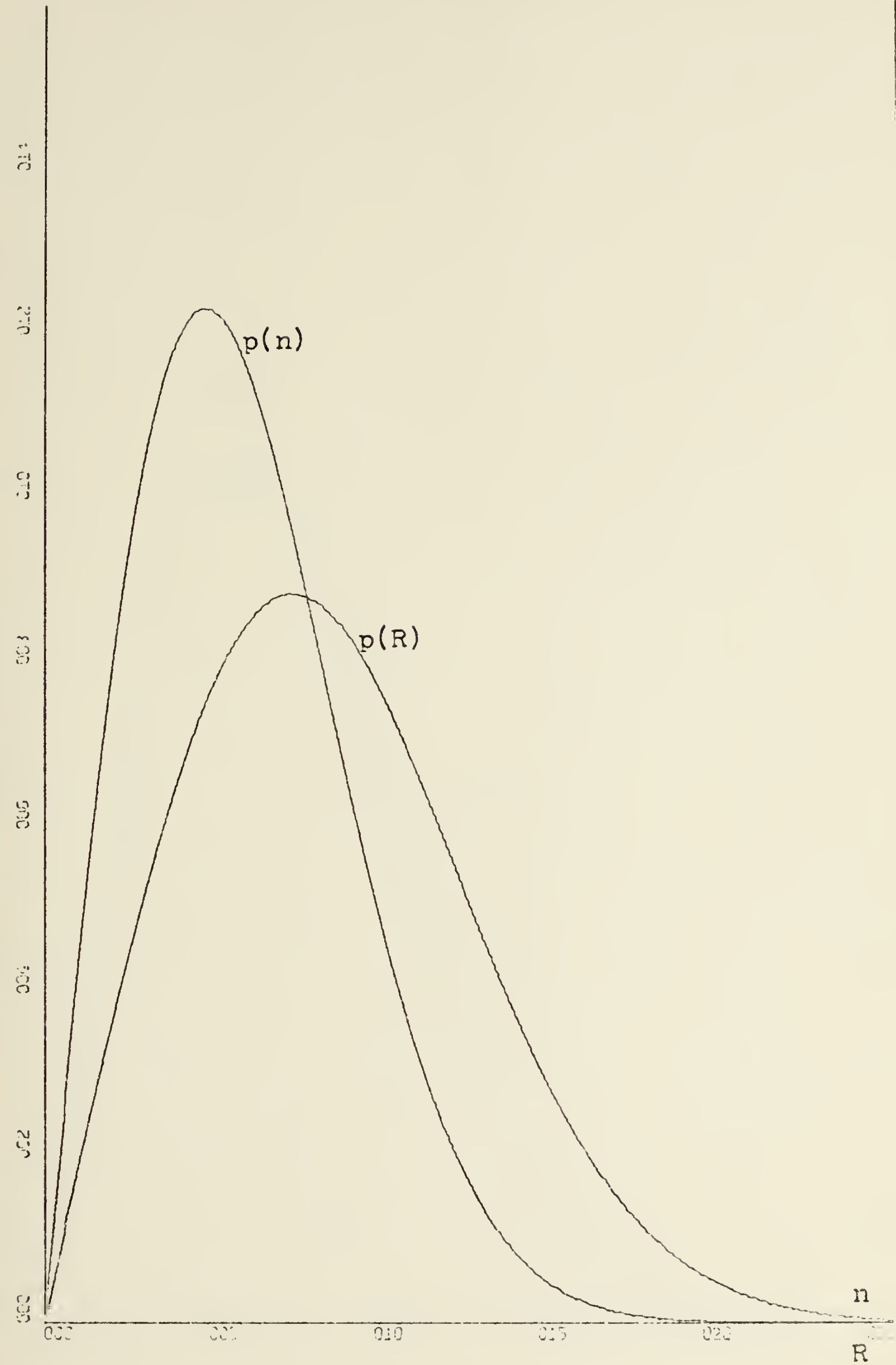

Fig. 16
Envelope PDF p(R), SNR=-3 db

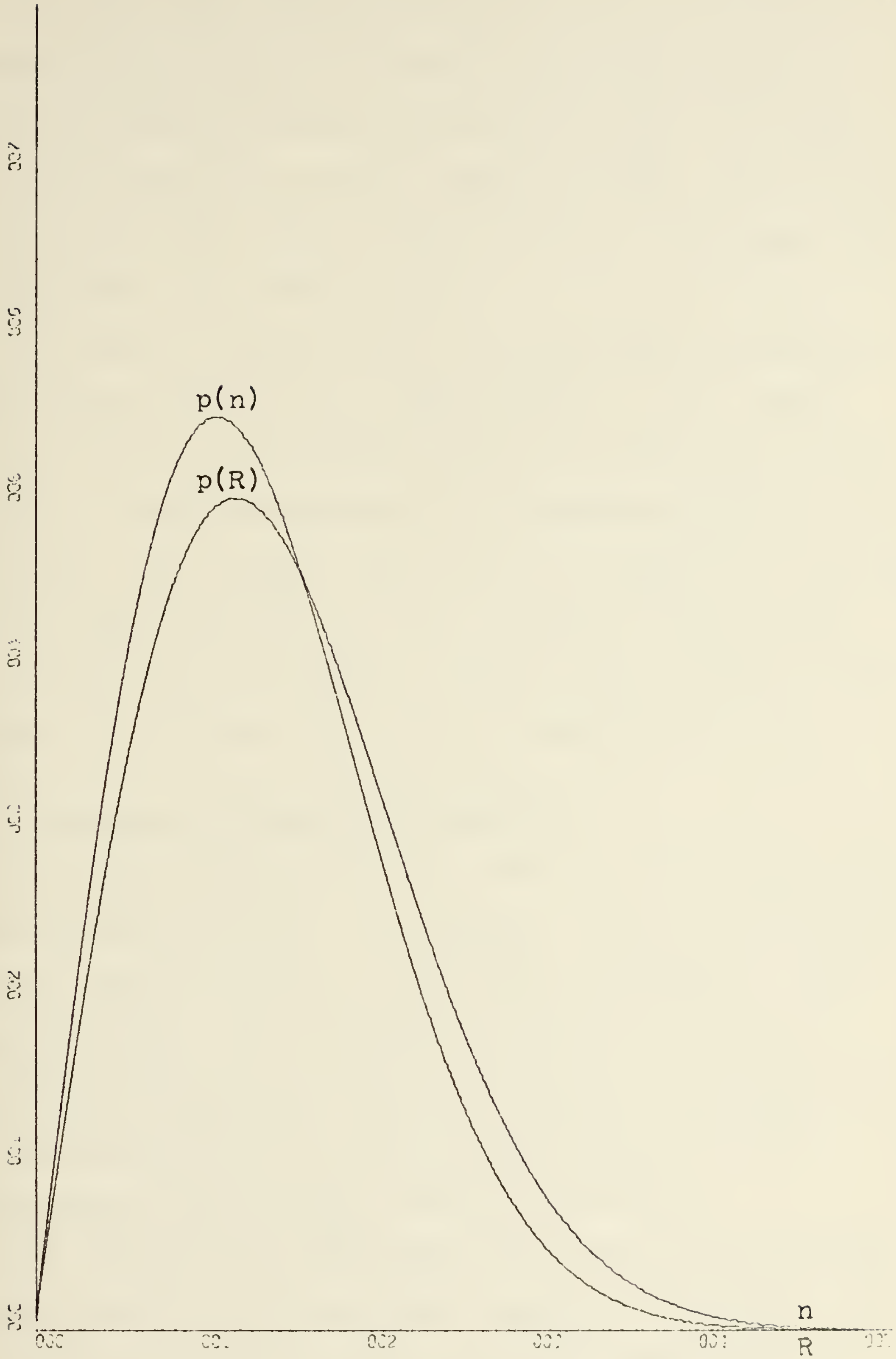

Fig. 17
Envelope PDF p(R), SNR=-10 db

Equation (16) can be used to calculate PD and PFA for various values of V_T. The results are then plotted on a Receiver Operating Characteristics (ROC) curve. In a ROC curve the ordinate is PD and the abscissa is PFA. The theoretically ideal operating point on such a curve is the upper left hand corner (PD = 1.0, PFA = 0.0).

As shown in Figures 18-22, the SSB signal is distinguishable from noise down to a SNR of about -3 db. These curves are useful in determining a measure of PD for a given SNR and PFA. Bandwidth changes do not appreciably alter the curves shown. Such changes, affecting the magnitude of envelope peaks, are significant only at high threshold values, and, thus, are confined to the region of the curve near PFA = 0.0. This aspect is discussed more fully in the next section.

3. Comparison of SSB and DSB Signals

In actual transmission the peaks of a SSB signal occur at a high repetition rate. Accordingly, a very high threshold can be set reducing the PFA to zero while retaining a small, but significant PD. An integration technique can then be employed by counting alarms over a relatively long period of time. Of interest is the relative effectiveness of this technique against equal power SSB and DSB signals.

In a square wave DSB signal, the envelope is constant. The information in the transmitted signal is contained in the phase of the carrier which reverses at the switching times. This type of modulation is commonly referred to as Phase

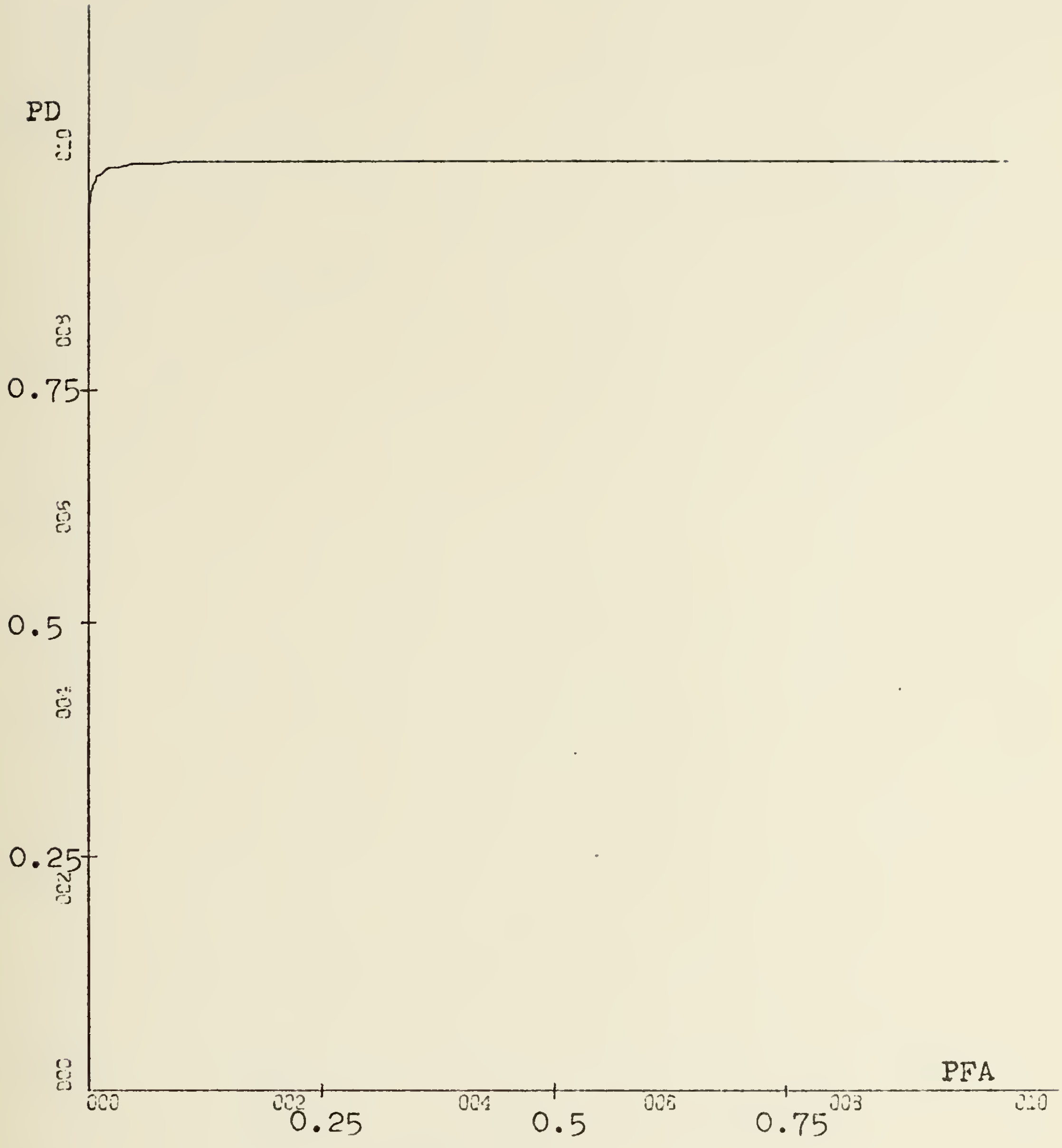

Fig. 18
ROC, SNR=+10 db, m = 7

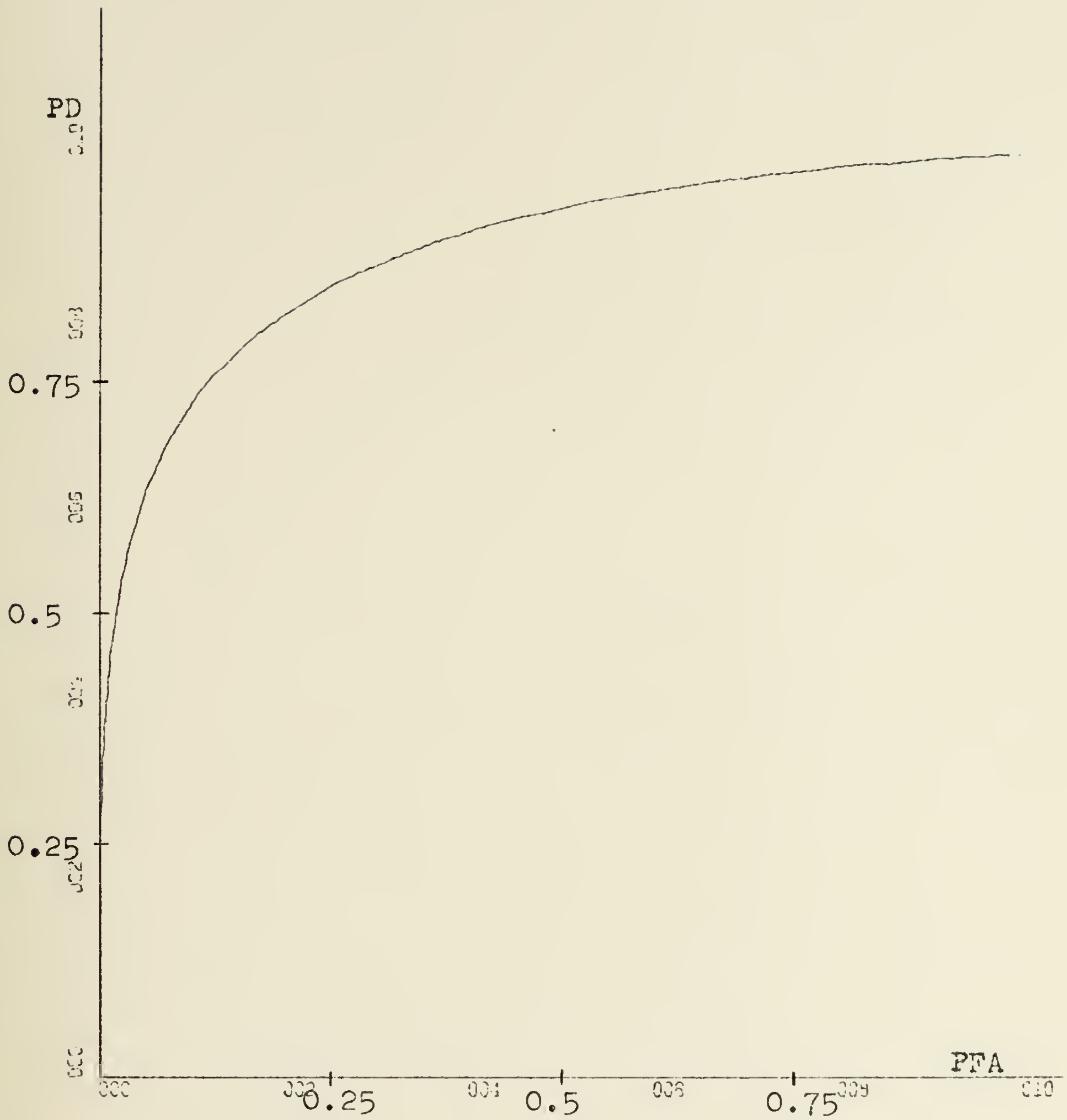

Fig. 19
ROC, SNR=+3 db, m = 7

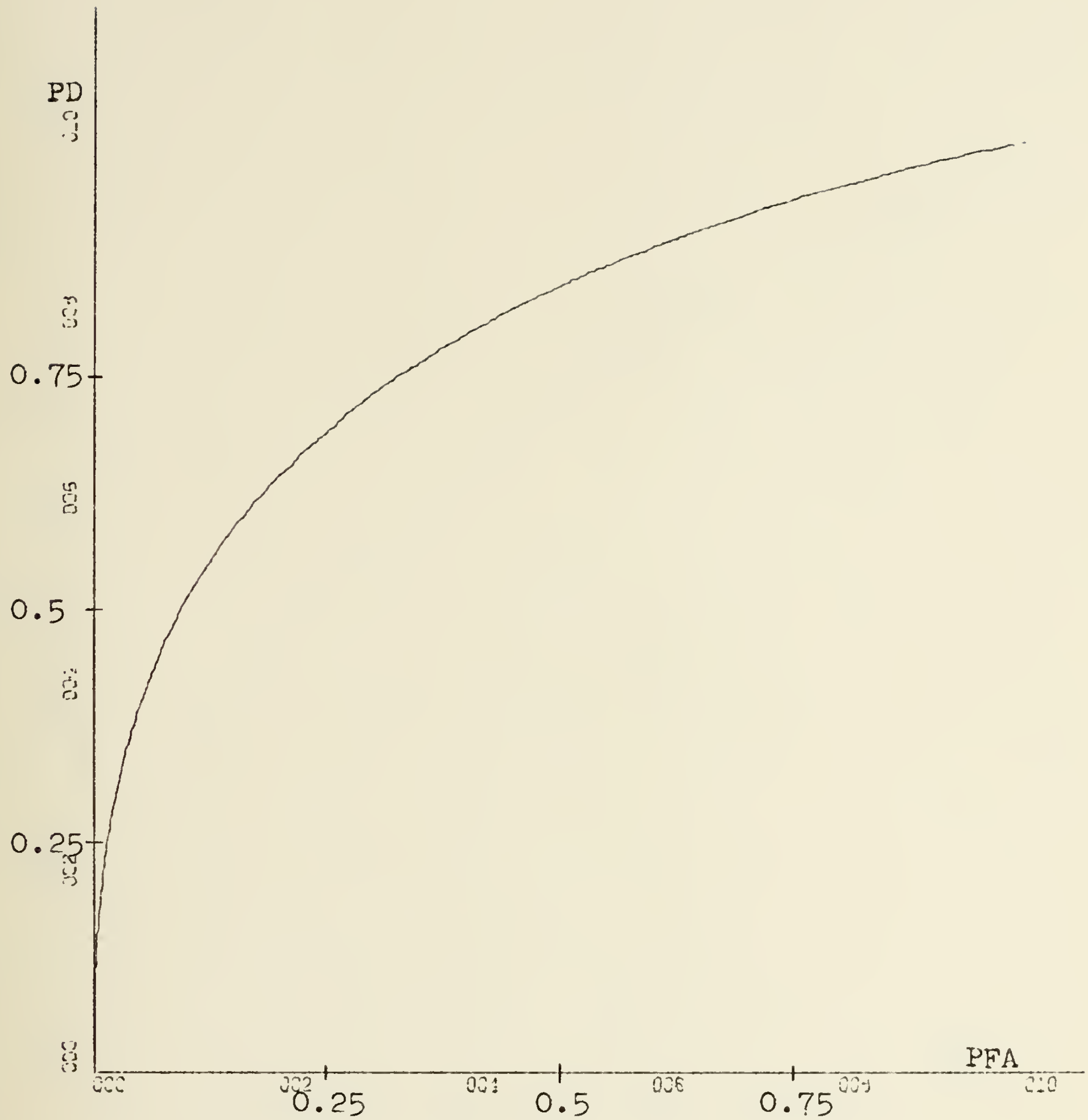

Fig. 20
ROC, SNR=0 db, m = 7

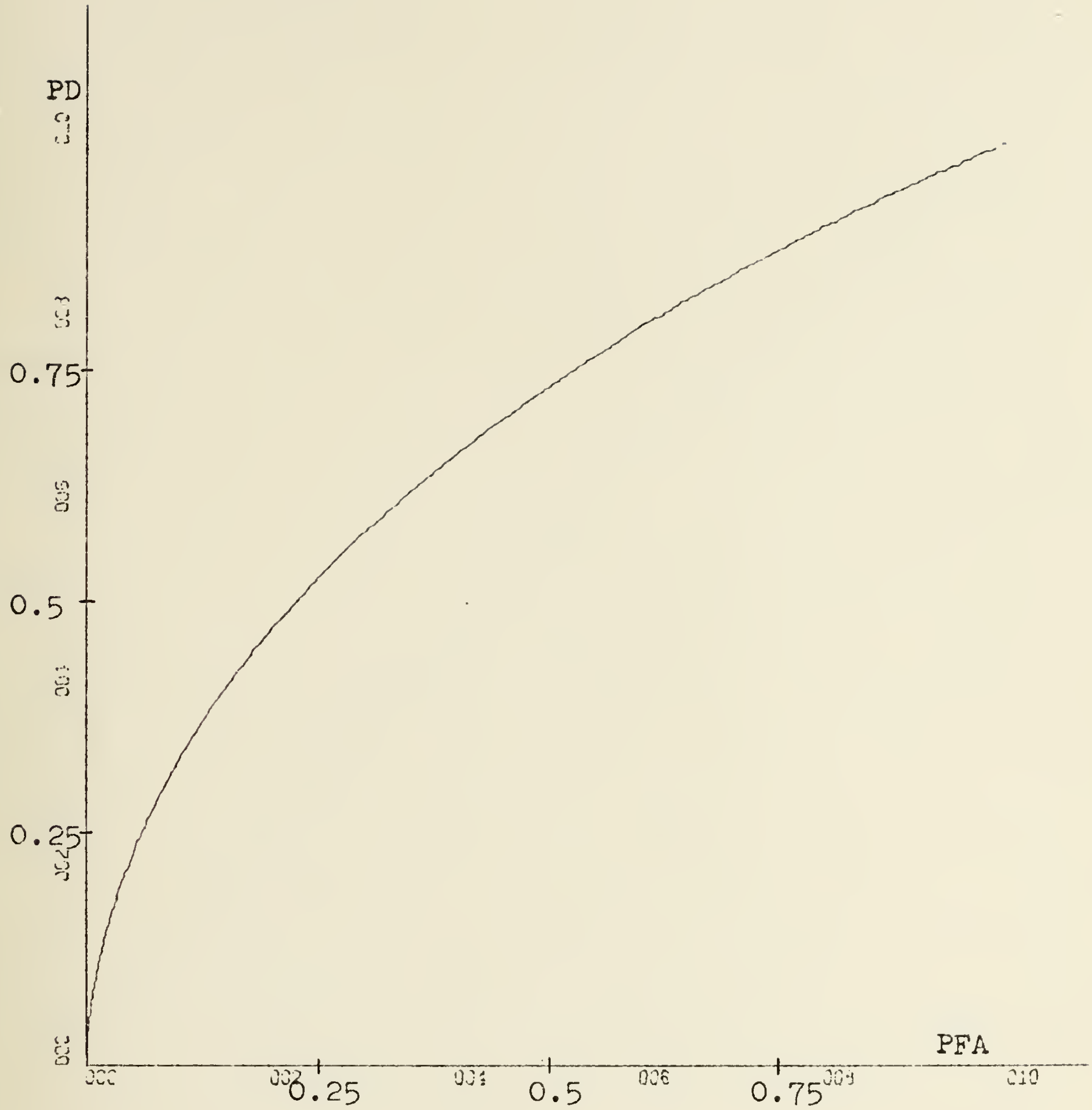

Fig. 21
ROC, SNR=-3 db, m = 7

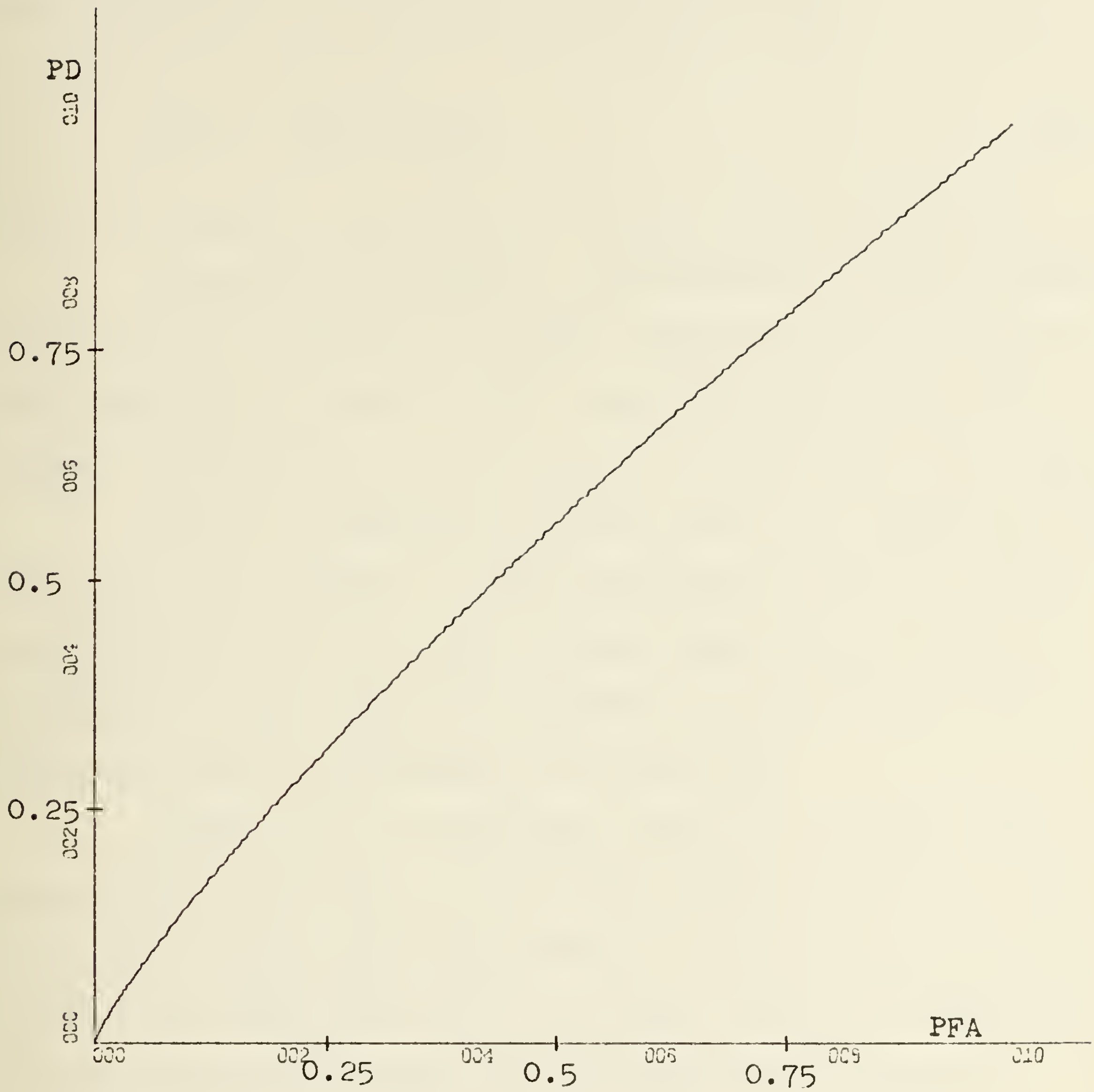

Fig. 22
ROC, SNR=-10 db, m = 7

Reversal modulation (PRM) or Phase Shift Keying (PSK). Since the envelope of the DSB signal equals the constant peak value of the sinusoid carrier, the envelope function E(t) for an equivalent power DSB signal can be written as,

$$E(t) = \sqrt{2}\, v_c(t)_{rms} \tag{20}$$

where $v_c(t)_{rms}$ is the SSB rms voltage obtained in Section II.

Using this expression, the analysis of the preceding section can be repeated to obtain the pdf of the envelope of the signal plus noise, PD, PFA, and ROC curves for a DSB signal. These can then be compared to those for a SSB signal.

Figure 23 shows the envelope pdf's for comparable SSB, DSB, and noise signals. Little difference is noted between the SSB and DSB curves until larger values of amplitude are reached, at which point the presence of peaks in the SSB waveform causes an increase in the SSB pdf.

Figure 24 is an expanded version of the ROC curve showing the difference in PD between SSB and DSB for large threshold values. The cross over of the two curves is indicative of the point where the SSB signal drops to a depressed value between peaks while the DSB envelope remains at a constant level.

Figures 23 and 24 indicate that the presence of peaks in the SSB envelope makes such signals more susceptible to detection by a high threshold-integration technique. At this point the effect of bandwidth can become significant.

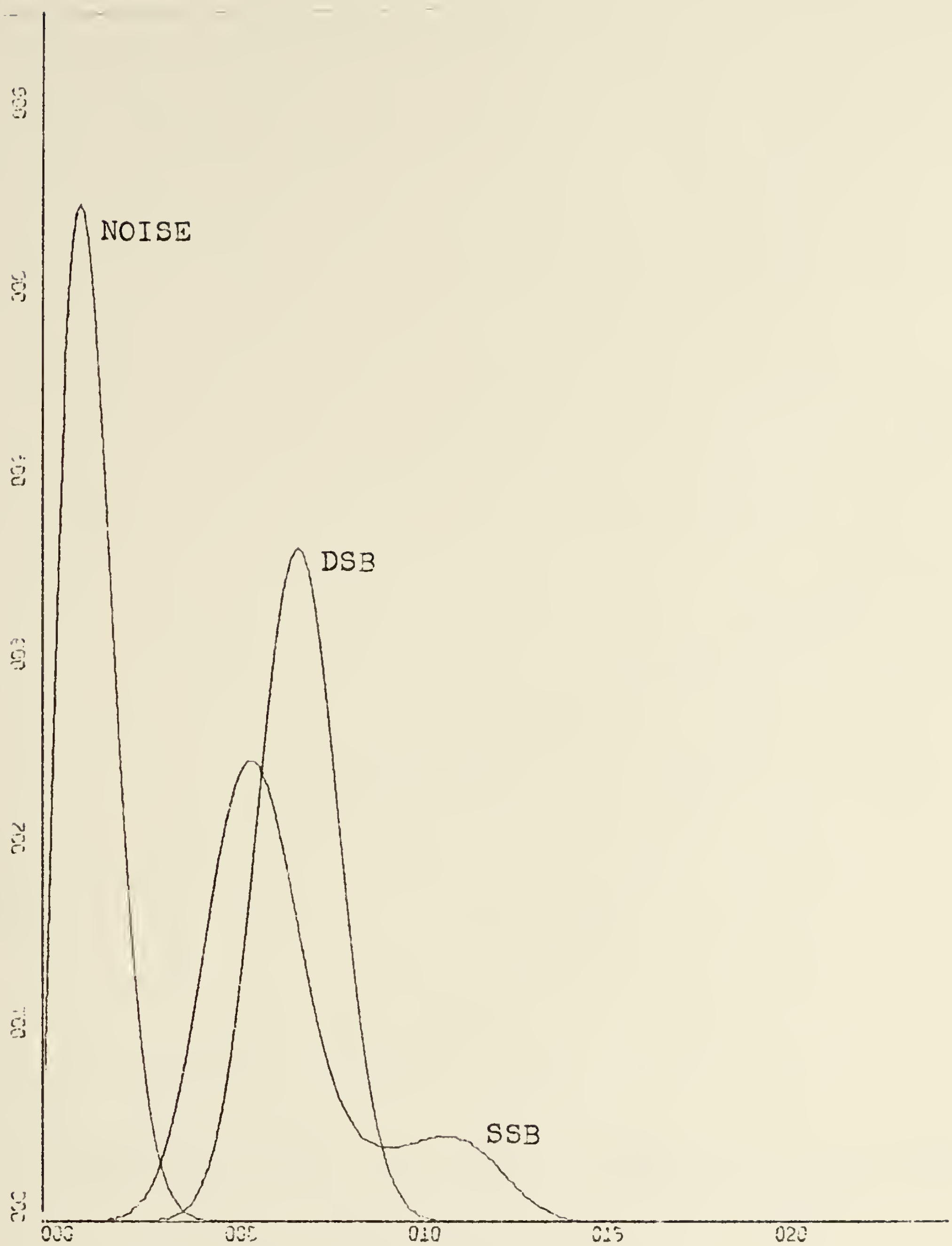

Fig. 23
Envelope PDF SSB, DSB, and Gaussian Noise, SNR=+10 db

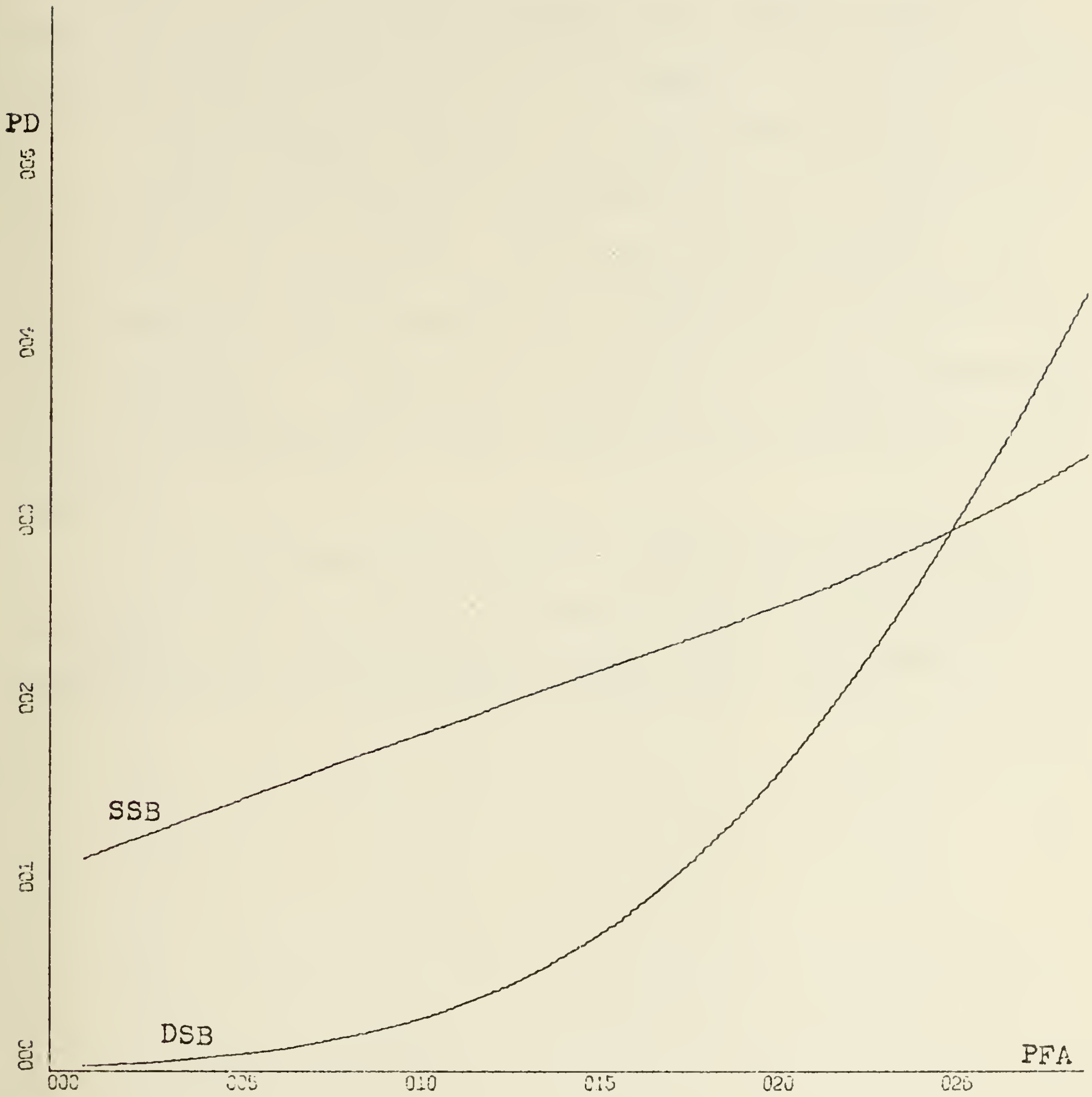

Fig. 24
Expanded ROC for SSB and DSB

To show this effect, selected sets of data are presented in Tables I through XII. Shown are PD for SSB and DSB signals. Bandwidth and SNR are variables (differ for different tables). This data can be usefully employed when design trade-off decisions are influenced by SNR, bandwidth and detectability.

For example, consider a system where the received SNR is 0 db and the bandwidth corresponds to m = 3. For a PFA less than one in 100,000, the PD for a SSB signal is .005, approximately double the PD for a DSB signal. If, however, the bandwidth is increased so that m = 15, the corresponding PD for a SSB signal is more than six times the PD for a DSB signal.

In this specific example, the acceptable risk of detection will strongly influence the choice of bandwidth used, as well as the choice of DSB or SSB modulation.

Table I
PD, SNR = +10 db, m = 3
(σ = 0.1488)

VT	PFA	PD(SSB)	PD(DSB)
0.60000	0.0	0.62502	0.75615
0.61000	0.0	0.60759	0.72536
0.62000	0.0	0.59007	0.69276
0.63000	0.0	0.57243	0.65857
0.64000	0.0	0.55466	0.62303
0.65000	0.0	0.53676	0.58643
0.66000	0.0	0.51870	0.54906
0.67000	0.0	0.50046	0.51125
0.68000	0.0	0.48204	0.47335
0.69000	0.0	0.46343	0.43569
0.70000	0.0	0.44462	0.39861
0.71000	0.0	0.42562	0.36243
0.72000	0.0	0.40645	0.32744
0.73000	0.0	0.38713	0.29391
0.74000	0.0	0.36768	0.26207
0.75000	0.0	0.34816	0.23209
0.76000	0.0	0.32860	0.20413
0.77000	0.0	0.30908	0.17829
0.78000	0.0	0.28965	0.15461
0.79000	0.0	0.27040	0.13310
0.80000	0.0	0.25140	0.11376
0.81000	0.0	0.23274	0.09651
0.82000	0.0	0.21450	0.08126
0.83000	0.0	0.19677	0.06791
0.84000	0.0	0.17963	0.05631
0.85000	0.0	0.16316	0.04634
0.86000	0.0	0.14743	0.03784
0.87000	0.0	0.13250	0.03066
0.88000	0.0	0.11842	0.02464
0.89000	0.0	0.10523	0.01965
0.90000	0.0	0.09296	0.01555
0.91000	0.0	0.08164	0.01220
0.92000	0.0	0.07125	0.00950
0.93000	0.0	0.06179	0.00733
0.94000	0.0	0.05325	0.00561
0.95000	0.0	0.04559	0.00426
0.96000	0.0	0.03877	0.00321
0.97000	0.0	0.03275	0.00240
0.98000	0.0	0.02748	0.00178
0.99000	0.0	0.02290	0.00130
1.00000	0.0	0.01894	0.00095
1.01000	0.0	0.01556	0.00068
1.02000	0.0	0.01269	0.00049
1.03000	0.0	0.01027	0.00035
1.04000	0.0	0.00826	0.00024
1.05000	0.0	0.00658	0.00017
1.06000	0.0	0.00521	0.00011
1.07000	0.0	0.00410	0.00008
1.08000	0.0	0.00319	0.00005
1.09000	0.0	0.00247	0.00003
1.10000	0.0	0.00189	0.00002
1.11000	0.0	0.00144	0.00001
1.12000	0.0	0.00109	0.00001
1.13000	0.0	0.00082	0.00000
1.14000	0.0	0.00061	0.00000
1.15000	0.0	0.00045	0.00000
1.16000	0.0	0.00032	0.0
1.17000	0.0	0.00023	0.0
1.18000	0.0	0.00017	0.0
1.19000	0.0	0.00012	0.0

Table II

PD, SNR = 0 db, m = 3

(σ = 0.474)

VT	PFA	PD(SSB)	PD(DSB)
1.20000	0.00154	0.09266	0.07914
1.21000	0.00138	0.08801	0.07470
1.22000	0.00124	0.08353	0.07046
1.23000	0.00111	0.07922	0.06640
1.24000	0.00099	0.07509	0.06253
1.25000	0.00089	0.07112	0.05884
1.26000	0.00079	0.06731	0.05532
1.27000	0.00071	0.06367	0.05198
1.28000	0.00063	0.06018	0.04879
1.29000	0.00056	0.05684	0.04577
1.30000	0.00050	0.05364	0.04289
1.31000	0.00045	0.05060	0.04017
1.32000	0.00040	0.04760	0.03759
1.33000	0.00035	0.04491	0.03515
1.34000	0.00031	0.04227	0.03284
1.35000	0.00028	0.03975	0.03065
1.36000	0.00024	0.03736	0.02859
1.37000	0.00022	0.03508	0.02665
1.38000	0.00019	0.03292	0.02482
1.39000	0.00017	0.03088	0.02309
1.40000	0.00015	0.02893	0.02147
1.41000	0.00013	0.02709	0.01994
1.42000	0.00012	0.02535	0.01851
1.43000	0.00010	0.02370	0.01717
1.44000	0.00009	0.02214	0.01591
1.45000	0.00008	0.02067	0.01473
1.46000	0.00007	0.01929	0.01363
1.47000	0.00006	0.01798	0.01260
1.48000	0.00005	0.01675	0.01164
1.49000	0.00005	0.01559	0.01074
1.50000	0.00004	0.01450	0.00990
1.51000	0.00003	0.01348	0.00912
1.52000	0.00003	0.01252	0.00840
1.53000	0.00003	0.01161	0.00773
1.54000	0.00002	0.01077	0.00710
1.55000	0.00002	0.00998	0.00652
1.56000	0.00002	0.00924	0.00598
1.57000	0.00001	0.00855	0.00548
1.58000	0.00001	0.00790	0.00502
1.59000	0.00001	0.00730	0.00460
1.60000	0.00001	0.00674	0.00420
1.61000	0.00001	0.00621	0.00384
1.62000	0.00001	0.00573	0.00351
1.63000	0.00001	0.00527	0.00320
1.64000	0.00000	0.00485	0.00291
1.65000	0.00000	0.00446	0.00265
1.66000	0.00000	0.00410	0.00241
1.67000	0.00000	0.00376	0.00219
1.68000	0.00000	0.00345	0.00199
1.69000	0.00000	0.00316	0.00181
1.70000	0.00000	0.00289	0.00164
1.71000	0.00000	0.00265	0.00148
1.72000	0.00000	0.00242	0.00134
1.73000	0.00000	0.00221	0.00121
1.74000	0.00000	0.00202	0.00109
1.75000	0.00000	0.00184	0.00099
1.76000	0.00000	0.00168	0.00089
1.77000	0.00000	0.00153	0.00080
1.78000	0.00000	0.00139	0.00072
1.79000	0.00000	0.00126	0.00065

Table III

PD, SNR = -10 db, m = 3

(σ = 1.488)

VT	PFA	PD(SSB)	PD(DSB)
4.21000	0.00034	0.00111	0.00106
4.22000	0.00033	0.00107	0.00102
4.23000	0.00031	0.00103	0.00098
4.24000	0.00030	0.00100	0.00095
4.25000	0.00029	0.00096	0.00092
4.26000	0.00028	0.00093	0.00088
4.27000	0.00027	0.00090	0.00085
4.28000	0.00025	0.00086	0.00082
4.29000	0.00024	0.00083	0.00079
4.30000	0.00023	0.00080	0.00076
4.31000	0.00022	0.00077	0.00074
4.32000	0.00021	0.00075	0.00071
4.33000	0.00021	0.00072	0.00068
4.34000	0.00020	0.00069	0.00066
4.35000	0.00019	0.00067	0.00063
4.36000	0.00018	0.00064	0.00061
4.37000	0.00017	0.00062	0.00059
4.38000	0.00017	0.00060	0.00057
4.39000	0.00016	0.00057	0.00054
4.40000	0.00015	0.00055	0.00052
4.41000	0.00014	0.00053	0.00050
4.42000	0.00014	0.00051	0.00048
4.43000	0.00013	0.00049	0.00047
4.44000	0.00013	0.00047	0.00045
4.45000	0.00012	0.00045	0.00043
4.46000	0.00011	0.00044	0.00041
4.47000	0.00011	0.00042	0.00040
4.48000	0.00010	0.00040	0.00038
4.49000	0.00010	0.00039	0.00036
4.50000	0.00009	0.00037	0.00035
4.51000	0.00009	0.00035	0.00033
4.52000	0.00009	0.00034	0.00032
4.53000	0.00008	0.00033	0.00031
4.54000	0.00008	0.00031	0.00029
4.55000	0.00007	0.00030	0.00028
4.56000	0.00007	0.00029	0.00027
4.57000	0.00007	0.00027	0.00026
4.58000	0.00006	0.00026	0.00025
4.59000	0.00006	0.00025	0.00024
4.60000	0.00006	0.00024	0.00022
4.61000	0.00005	0.00023	0.00021
4.62000	0.00005	0.00022	0.00020
4.63000	0.00005	0.00021	0.00019
4.64000	0.00004	0.00020	0.00019
4.65000	0.00004	0.00019	0.00018
4.66000	0.00004	0.00018	0.00017
4.67000	0.00004	0.00017	0.00016
4.68000	0.00003	0.00016	0.00015
4.69000	0.00003	0.00015	0.00014
4.70000	0.00003	0.00014	0.00014
4.71000	0.00003	0.00014	0.00013
4.72000	0.00002	0.00013	0.00012
4.73000	0.00002	0.00012	0.00011
4.74000	0.00002	0.00012	0.00011
4.75000	0.00002	0.00011	0.00010
4.76000	0.00002	0.00010	0.00010
4.77000	0.00002	0.00010	0.00009
4.78000	0.00001	0.00009	0.00008
4.79000	0.00001	0.00008	0.00008
4.80000	0.00001	0.00008	0.00007

Table IV

PD,SNR = +10 db, m = 7

(σ = 0.1535)

VT	PFA	PD(SSB)	PD(DSB)
0.60000	0.00000	0.53624	0.80040
0.61000	0.0	0.51407	0.77347
0.62000	0.0	0.49269	0.74461
0.63000	0.0	0.47216	0.71395
0.64000	0.0	0.45256	0.68164
0.65000	0.0	0.43393	0.64789
0.66000	0.0	0.41631	0.61294
0.67000	0.0	0.39969	0.57706
0.68000	0.0	0.38408	0.54052
0.69000	0.0	0.36945	0.50365
0.70000	0.0	0.35575	0.46675
0.71000	0.0	0.34295	0.43013
0.72000	0.0	0.33099	0.39412
0.73000	0.0	0.31980	0.35899
0.74000	0.0	0.30931	0.32501
0.75000	0.0	0.29946	0.29244
0.76000	0.0	0.29016	0.26147
0.77000	0.0	0.28136	0.23229
0.78000	0.0	0.27297	0.20501
0.79000	0.0	0.26494	0.17974
0.80000	0.0	0.25721	0.15652
0.81000	0.0	0.24970	0.13537
0.82000	0.0	0.24238	0.11627
0.83000	0.0	0.23520	0.09916
0.84000	0.0	0.22811	0.08398
0.85000	0.0	0.22107	0.07061
0.86000	0.0	0.21405	0.05894
0.87000	0.0	0.20703	0.04884
0.88000	0.0	0.19998	0.04018
0.89000	0.0	0.19289	0.03280
0.90000	0.0	0.18573	0.02659
0.91000	0.0	0.17851	0.02138
0.92000	0.0	0.17122	0.01707
0.93000	0.0	0.16386	0.01352
0.94000	0.0	0.15644	0.01063
0.95000	0.0	0.14895	0.00829
0.96000	0.0	0.14143	0.00642
0.97000	0.0	0.13389	0.00493
0.98000	0.0	0.12634	0.00376
0.99000	0.0	0.11881	0.00284
1.00000	0.0	0.11133	0.00213
1.01000	0.0	0.10393	0.00158
1.02000	0.0	0.09664	0.00117
1.03000	0.0	0.08948	0.00085
1.04000	0.0	0.08250	0.00062
1.05000	0.0	0.07572	0.00044
1.06000	0.0	0.06918	0.00032
1.07000	0.0	0.06290	0.00022
1.08000	0.0	0.05690	0.00015
1.09000	0.0	0.05121	0.00011
1.10000	0.0	0.04584	0.00007
1.11000	0.0	0.04081	0.00005
1.12000	0.0	0.03614	0.00003
1.13000	0.0	0.03181	0.00002
1.14000	0.0	0.02784	0.00001
1.15000	0.0	0.02422	0.00001
1.16000	0.0	0.02095	0.00000
1.17000	0.0	0.01800	0.00000
1.18000	0.0	0.01537	0.00000
1.19000	0.0	0.01304	0.0

Table V
PD, SNR = 0 db, m = 7
(σ = 0.486)

VT	PFA	PD(SSB)	PD(DSB)
1.20000	0.00214	0.11668	0.09426
1.21000	0.00194	0.11184	0.08934
1.22000	0.00174	0.10716	0.08459
1.23000	0.00157	0.10262	0.08004
1.24000	0.00142	0.09822	0.07568
1.25000	0.00127	0.09397	0.07150
1.26000	0.00114	0.08986	0.06750
1.27000	0.00103	0.08588	0.06368
1.28000	0.00092	0.08204	0.06003
1.29000	0.00083	0.07833	0.05655
1.30000	0.00074	0.07475	0.05322
1.31000	0.00066	0.07130	0.05006
1.32000	0.00059	0.06797	0.04705
1.33000	0.00053	0.06477	0.04419
1.34000	0.00047	0.06168	0.04147
1.35000	0.00042	0.05871	0.03888
1.36000	0.00037	0.05586	0.03644
1.37000	0.00033	0.05311	0.03411
1.38000	0.00030	0.05047	0.03192
1.39000	0.00026	0.04794	0.02984
1.40000	0.00023	0.04551	0.02787
1.41000	0.00021	0.04318	0.02602
1.42000	0.00018	0.04095	0.02427
1.43000	0.00016	0.03881	0.02262
1.44000	0.00014	0.03676	0.02106
1.45000	0.00013	0.03480	0.01960
1.46000	0.00011	0.03293	0.01823
1.47000	0.00010	0.03114	0.01693
1.48000	0.00009	0.02943	0.01572
1.49000	0.00008	0.02779	0.01459
1.50000	0.00007	0.02624	0.01352
1.51000	0.00006	0.02475	0.01252
1.52000	0.00005	0.02334	0.01159
1.53000	0.00005	0.02199	0.01072
1.54000	0.00004	0.02071	0.00990
1.55000	0.00003	0.01950	0.00914
1.56000	0.00003	0.01834	0.00844
1.57000	0.00003	0.01724	0.00778
1.58000	0.00002	0.01620	0.00716
1.59000	0.00002	0.01521	0.00659
1.60000	0.00002	0.01427	0.00606
1.61000	0.00001	0.01338	0.00557
1.62000	0.00001	0.01254	0.00512
1.63000	0.00001	0.01175	0.00469
1.64000	0.00001	0.01099	0.00430
1.65000	0.00001	0.01028	0.00394
1.66000	0.00001	0.00961	0.00361
1.67000	0.00001	0.00898	0.00330
1.68000	0.00001	0.00838	0.00301
1.69000	0.00000	0.00782	0.00275
1.70000	0.00000	0.00729	0.00251
1.71000	0.00000	0.00679	0.00229
1.72000	0.00000	0.00633	0.00208
1.73000	0.00000	0.00589	0.00189
1.74000	0.00000	0.00547	0.00172
1.75000	0.00000	0.00509	0.00156
1.76000	0.00000	0.00472	0.00142
1.77000	0.00000	0.00438	0.00129
1.78000	0.00000	0.00406	0.00117
1.79000	0.00000	0.00377	0.00105

Table VI
PD, SNR = -10 db, m = 7
(σ = 1.535)

VT	PFA	PD(SSB)	PD(DSB)
4.21000	0.00052	0.00161	0.00150
4.22000	0.00050	0.00156	0.00145
4.23000	0.00048	0.00151	0.00140
4.24000	0.00046	0.00146	0.00135
4.25000	0.00044	0.00141	0.00131
4.26000	0.00043	0.00136	0.00126
4.27000	0.00041	0.00132	0.00122
4.28000	0.00040	0.00127	0.00118
4.29000	0.00038	0.00123	0.00114
4.30000	0.00037	0.00119	0.00110
4.31000	0.00035	0.00115	0.00106
4.32000	0.00034	0.00111	0.00102
4.33000	0.00032	0.00107	0.00099
4.34000	0.00031	0.00103	0.00095
4.35000	0.00030	0.00100	0.00092
4.36000	0.00029	0.00096	0.00089
4.37000	0.00028	0.00093	0.00085
4.38000	0.00027	0.00090	0.00082
4.39000	0.00026	0.00086	0.00079
4.40000	0.00024	0.00083	0.00076
4.41000	0.00023	0.00080	0.00074
4.42000	0.00023	0.00077	0.00071
4.43000	0.00022	0.00074	0.00068
4.44000	0.00021	0.00072	0.00066
4.45000	0.00020	0.00069	0.00063
4.46000	0.00019	0.00066	0.00061
4.47000	0.00018	0.00064	0.00058
4.48000	0.00017	0.00061	0.00056
4.49000	0.00017	0.00059	0.00054
4.50000	0.00016	0.00057	0.00052
4.51000	0.00015	0.00054	0.00050
4.52000	0.00015	0.00052	0.00048
4.53000	0.00014	0.00050	0.00046
4.54000	0.00013	0.00048	0.00044
4.55000	0.00013	0.00046	0.00042
4.56000	0.00012	0.00044	0.00040
4.57000	0.00012	0.00042	0.00039
4.58000	0.00011	0.00041	0.00037
4.59000	0.00011	0.00039	0.00035
4.60000	0.00010	0.00037	0.00034
4.61000	0.00010	0.00036	0.00032
4.62000	0.00009	0.00034	0.00031
4.63000	0.00009	0.00032	0.00029
4.64000	0.00008	0.00031	0.00028
4.65000	0.00008	0.00029	0.00027
4.66000	0.00007	0.00028	0.00025
4.67000	0.00007	0.00027	0.00024
4.68000	0.00007	0.00025	0.00023
4.69000	0.00006	0.00024	0.00022
4.70000	0.00006	0.00023	0.00021
4.71000	0.00006	0.00022	0.00020
4.72000	0.00005	0.00021	0.00019
4.73000	0.00005	0.00019	0.00018
4.74000	0.00005	0.00018	0.00017
4.75000	0.00004	0.00017	0.00016
4.76000	0.00004	0.00016	0.00015
4.77000	0.00004	0.00015	0.00014
4.78000	0.00004	0.00014	0.00013
4.79000	0.00003	0.00014	0.00012
4.80000	0.00003	0.00013	0.00011

Table VII

PD, SNR = +10 db, m = 11

(σ = 0.1550)

VT	PFA	PD(SSB)	PD(DSB)
0.60000	0.00000	0.53808	0.81360
0.61000	0.0	0.51392	0.78795
0.62000	0.0	0.49032	0.76035
0.63000	0.0	0.46741	0.73090
0.64000	0.0	0.44527	0.69974
0.65000	0.0	0.42401	0.66706
0.66000	0.0	0.40368	0.63305
0.67000	0.0	0.38434	0.59797
0.68000	0.0	0.36602	0.56209
0.69000	0.0	0.34876	0.52570
0.70000	0.0	0.33255	0.48910
0.71000	0.0	0.31739	0.45259
0.72000	0.0	0.30327	0.41648
0.73000	0.0	0.29014	0.38106
0.74000	0.0	0.27798	0.34662
0.75000	0.0	0.26674	0.31341
0.76000	0.0	0.25636	0.28165
0.77000	0.0	0.24679	0.25153
0.78000	0.0	0.23796	0.22320
0.79000	0.0	0.22981	0.19679
0.80000	0.0	0.22229	0.17237
0.81000	0.0	0.21532	0.14997
0.82000	0.0	0.20885	0.12961
0.83000	0.0	0.20282	0.11124
0.84000	0.0	0.19718	0.09482
0.85000	0.0	0.19186	0.08026
0.86000	0.0	0.18683	0.06746
0.87000	0.0	0.18204	0.05630
0.88000	0.0	0.17745	0.04664
0.89000	0.0	0.17302	0.03837
0.90000	0.0	0.16872	0.03133
0.91000	0.0	0.16452	0.02539
0.92000	0.0	0.16039	0.02043
0.93000	0.0	0.15631	0.01632
0.94000	0.0	0.15226	0.01293
0.95000	0.0	0.14822	0.01017
0.96000	0.0	0.14417	0.00794
0.97000	0.0	0.14011	0.00615
0.98000	0.0	0.13601	0.00473
0.99000	0.0	0.13188	0.00361
1.00000	0.0	0.12770	0.00273
1.01000	0.0	0.12347	0.00205
1.02000	0.0	0.11918	0.00153
1.03000	0.0	0.11483	0.00113
1.04000	0.0	0.11042	0.00083
1.05000	0.0	0.10595	0.00060
1.06000	0.0	0.10143	0.00043
1.07000	0.0	0.09687	0.00031
1.08000	0.0	0.09226	0.00022
1.09000	0.0	0.08763	0.00015
1.10000	0.0	0.08298	0.00010
1.11000	0.0	0.07832	0.00007
1.12000	0.0	0.07368	0.00005
1.13000	0.0	0.06907	0.00003
1.14000	0.0	0.06451	0.00002
1.15000	0.0	0.06001	0.00001
1.16000	0.0	0.05560	0.00001
1.17000	0.0	0.05130	0.00000
1.18000	0.0	0.04712	0.00000
1.19000	0.0	0.04308	0.00000

Table VIII

PD, SNR = 0 db, m = 11

(σ = 0.490)

VT	PFA	PD(SSB)	PD(DSB)
1.20000	0.00238	0.12434	0.09962
1.21000	0.00215	0.11960	0.09451
1.22000	0.00195	0.11500	0.08960
1.23000	0.00176	0.11053	0.08488
1.24000	0.00158	0.10620	0.08036
1.25000	0.00143	0.10200	0.07602
1.26000	0.00129	0.09793	0.07186
1.27000	0.00116	0.09399	0.06788
1.28000	0.00104	0.09017	0.06407
1.29000	0.00093	0.08647	0.06043
1.30000	0.00084	0.08290	0.05696
1.31000	0.00075	0.07944	0.05365
1.32000	0.00067	0.07609	0.05049
1.33000	0.00060	0.07286	0.04749
1.34000	0.00054	0.06974	0.04463
1.35000	0.00048	0.06672	0.04191
1.36000	0.00043	0.06381	0.03932
1.37000	0.00038	0.06100	0.03687
1.38000	0.00034	0.05829	0.03455
1.39000	0.00030	0.05568	0.03235
1.40000	0.00027	0.05316	0.03026
1.41000	0.00024	0.05074	0.02829
1.42000	0.00021	0.04840	0.02643
1.43000	0.00019	0.04615	0.02467
1.44000	0.00017	0.04399	0.02301
1.45000	0.00015	0.04191	0.02145
1.46000	0.00013	0.03991	0.01997
1.47000	0.00012	0.03799	0.01859
1.48000	0.00010	0.03614	0.01728
1.49000	0.00009	0.03437	0.01606
1.50000	0.00008	0.03267	0.01491
1.51000	0.00007	0.03104	0.01383
1.52000	0.00006	0.02948	0.01283
1.53000	0.00005	0.02798	0.01188
1.54000	0.00005	0.02654	0.01100
1.55000	0.00004	0.02517	0.01017
1.56000	0.00004	0.02385	0.00940
1.57000	0.00003	0.02260	0.00868
1.58000	0.00003	0.02139	0.00801
1.59000	0.00002	0.02025	0.00739
1.60000	0.00002	0.01915	0.00680
1.61000	0.00002	0.01810	0.00626
1.62000	0.00002	0.01710	0.00576
1.63000	0.00001	0.01615	0.00530
1.64000	0.00001	0.01524	0.00486
1.65000	0.00001	0.01438	0.00446
1.66000	0.00001	0.01355	0.00409
1.67000	0.00001	0.01277	0.00375
1.68000	0.00001	0.01203	0.00343
1.69000	0.00001	0.01132	0.00314
1.70000	0.00000	0.01065	0.00287
1.71000	0.00000	0.01001	0.00262
1.72000	0.00000	0.00940	0.00239
1.73000	0.00000	0.00883	0.00218
1.74000	0.00000	0.00829	0.00199
1.75000	0.00000	0.00777	0.00181
1.76000	0.00000	0.00728	0.00164
1.77000	0.00000	0.00682	0.00149
1.78000	0.00000	0.00639	0.00136
1.79000	0.00000	0.00598	0.00123

Table IX
PD, SNR = -10 db, m = 11
(σ = 1.550)

VT	PFA	PD(SSB)	PD(DSB)
4.21000	0.00059	0.00184	0.00167
4.22000	0.00057	0.00178	0.00162
4.23000	0.00054	0.00172	0.00156
4.24000	0.00052	0.00166	0.00151
4.25000	0.00051	0.00161	0.00146
4.26000	0.00049	0.00156	0.00141
4.27000	0.00047	0.00151	0.00137
4.28000	0.00045	0.00146	0.00132
4.29000	0.00043	0.00141	0.00127
4.30000	0.00042	0.00136	0.00123
4.31000	0.00040	0.00132	0.00119
4.32000	0.00039	0.00127	0.00115
4.33000	0.00037	0.00123	0.00111
4.34000	0.00036	0.00119	0.00107
4.35000	0.00034	0.00115	0.00103
4.36000	0.00033	0.00111	0.00100
4.37000	0.00032	0.00107	0.00096
4.38000	0.00031	0.00103	0.00093
4.39000	0.00029	0.00099	0.00089
4.40000	0.00028	0.00096	0.00086
4.41000	0.00027	0.00092	0.00083
4.42000	0.00026	0.00089	0.00080
4.43000	0.00025	0.00086	0.00077
4.44000	0.00024	0.00083	0.00074
4.45000	0.00023	0.00080	0.00071
4.46000	0.00022	0.00077	0.00069
4.47000	0.00021	0.00074	0.00066
4.48000	0.00020	0.00071	0.00063
4.49000	0.00019	0.00068	0.00061
4.50000	0.00018	0.00066	0.00059
4.51000	0.00018	0.00063	0.00056
4.52000	0.00017	0.00061	0.00054
4.53000	0.00016	0.00058	0.00052
4.54000	0.00015	0.00056	0.00050
4.55000	0.00015	0.00054	0.00048
4.56000	0.00014	0.00052	0.00046
4.57000	0.00013	0.00049	0.00044
4.58000	0.00013	0.00047	0.00042
4.59000	0.00012	0.00045	0.00040
4.60000	0.00012	0.00043	0.00038
4.61000	0.00011	0.00042	0.00037
4.62000	0.00011	0.00040	0.00035
4.63000	0.00010	0.00038	0.00033
4.64000	0.00010	0.00036	0.00032
4.65000	0.00009	0.00035	0.00030
4.66000	0.00009	0.00033	0.00029
4.67000	0.00008	0.00031	0.00028
4.68000	0.00008	0.00030	0.00026
4.69000	0.00007	0.00028	0.00025
4.70000	0.00007	0.00027	0.00024
4.71000	0.00007	0.00026	0.00022
4.72000	0.00006	0.00024	0.00021
4.73000	0.00006	0.00023	0.00020
4.74000	0.00006	0.00022	0.00019
4.75000	0.00005	0.00020	0.00018
4.76000	0.00005	0.00019	0.00017
4.77000	0.00005	0.00018	0.00016
4.78000	0.00004	0.00017	0.00015
4.79000	0.00004	0.00016	0.00014
4.80000	0.00004	0.00015	0.00013

Table X
PD, SNR = +10 db, m = 15
(σ = 0.1555)

VT	PFA	PD(SSB)	PD(DSB)
0.60000	0.00000	0.54265	0.81991
0.61000	0.00000	0.51851	0.79489
0.62000	0.0	0.49483	0.76791
0.63000	0.0	0.47171	0.73907
0.64000	0.0	0.44926	0.70849
0.65000	0.0	0.42753	0.67635
0.66000	0.0	0.40659	0.64284
0.67000	0.0	0.38650	0.60819
0.68000	0.0	0.36730	0.57267
0.69000	0.0	0.34901	0.53655
0.70000	0.0	0.33165	0.50013
0.71000	0.0	0.31523	0.46372
0.72000	0.0	0.29976	0.42761
0.73000	0.0	0.28521	0.39210
0.74000	0.0	0.27159	0.35747
0.75000	0.0	0.25886	0.32398
0.76000	0.0	0.24699	0.29186
0.77000	0.0	0.23596	0.26132
0.78000	0.0	0.22573	0.23250
0.79000	0.0	0.21625	0.20555
0.80000	0.0	0.20750	0.18055
0.81000	0.0	0.19941	0.15754
0.82000	0.0	0.19196	0.13656
0.83000	0.0	0.18508	0.11757
0.84000	0.0	0.17874	0.10054
0.85000	0.0	0.17289	0.08538
0.86000	0.0	0.16747	0.07200
0.87000	0.0	0.16246	0.06029
0.88000	0.0	0.15780	0.05013
0.89000	0.0	0.15346	0.04138
0.90000	0.0	0.14939	0.03392
0.91000	0.0	0.14556	0.02759
0.92000	0.0	0.14194	0.02229
0.93000	0.0	0.13849	0.01787
0.94000	0.0	0.13519	0.01422
0.95000	0.0	0.13201	0.01123
0.96000	0.0	0.12893	0.00881
0.97000	0.0	0.12593	0.00685
0.98000	0.0	0.12298	0.00529
0.99000	0.0	0.12008	0.00405
1.00000	0.0	0.11721	0.00308
1.01000	0.0	0.11436	0.00233
1.02000	0.0	0.11151	0.00174
1.03000	0.0	0.10866	0.00129
1.04000	0.0	0.10580	0.00095
1.05000	0.0	0.10292	0.00070
1.06000	0.0	0.10001	0.00050
1.07000	0.0	0.09707	0.00036
1.08000	0.0	0.09410	0.00026
1.09000	0.0	0.09109	0.00018
1.10000	0.0	0.08804	0.00013
1.11000	0.0	0.08494	0.00009
1.12000	0.0	0.08181	0.00006
1.13000	0.0	0.07863	0.00004
1.14000	0.0	0.07541	0.00002
1.15000	0.0	0.07215	0.00002
1.16000	0.0	0.06886	0.00001
1.17000	0.0	0.06555	0.00001
1.18000	0.0	0.06222	0.00000
1.19000	0.0	0.05887	0.00000

Table XI

PD, SNR = 0 db, m = 15

(σ = 0.492)

VT	PFA	PD(SSB)	PD(DSB)
1.20000	0.00251	0.12621	0.10233
1.21000	0.00227	0.12157	0.09714
1.22000	0.00205	0.11706	0.09214
1.23000	0.00185	0.11268	0.08735
1.24000	0.00167	0.10844	0.08274
1.25000	0.00151	0.10433	0.07832
1.26000	0.00136	0.10034	0.07408
1.27000	0.00122	0.09648	0.07002
1.28000	0.00110	0.09274	0.06614
1.29000	0.00099	0.08911	0.06243
1.30000	0.00089	0.08560	0.05888
1.31000	0.00080	0.08221	0.05549
1.32000	0.00072	0.07892	0.05226
1.33000	0.00064	0.07575	0.04918
1.34000	0.00057	0.07268	0.04625
1.35000	0.00051	0.06971	0.04346
1.36000	0.00046	0.06684	0.04081
1.37000	0.00041	0.06407	0.03830
1.38000	0.00037	0.06139	0.03591
1.39000	0.00032	0.05880	0.03364
1.40000	0.00029	0.05631	0.03150
1.41000	0.00026	0.05390	0.02947
1.42000	0.00023	0.05158	0.02755
1.43000	0.00020	0.04934	0.02574
1.44000	0.00018	0.04718	0.02402
1.45000	0.00016	0.04510	0.02241
1.46000	0.00014	0.04310	0.02088
1.47000	0.00013	0.04117	0.01945
1.48000	0.00011	0.03931	0.01810
1.49000	0.00010	0.03752	0.01683
1.50000	0.00009	0.03580	0.01564
1.51000	0.00008	0.03415	0.01452
1.52000	0.00007	0.03256	0.01348
1.53000	0.00006	0.03103	0.01249
1.54000	0.00005	0.02956	0.01157
1.55000	0.00005	0.02814	0.01071
1.56000	0.00004	0.02679	0.00991
1.57000	0.00003	0.02549	0.00916
1.58000	0.00003	0.02424	0.00846
1.59000	0.00003	0.02304	0.00781
1.60000	0.00002	0.02189	0.00720
1.61000	0.00002	0.02079	0.00663
1.62000	0.00002	0.01974	0.00611
1.63000	0.00002	0.01873	0.00562
1.64000	0.00001	0.01776	0.00516
1.65000	0.00001	0.01684	0.00474
1.66000	0.00001	0.01595	0.00435
1.67000	0.00001	0.01511	0.00399
1.68000	0.00001	0.01430	0.00366
1.69000	0.00001	0.01353	0.00335
1.70000	0.00001	0.01279	0.00306
1.71000	0.00000	0.01209	0.00280
1.72000	0.00000	0.01142	0.00256
1.73000	0.00000	0.01078	0.00234
1.74000	0.00000	0.01017	0.00213
1.75000	0.00000	0.00959	0.00194
1.76000	0.00000	0.00904	0.00177
1.77000	0.00000	0.00852	0.00161
1.78000	0.00000	0.00802	0.00146
1.79000	0.00000	0.00755	0.00133

Table XII
PD, SNR = -10 db, m = 15
(σ = 1.555)

VT	PFA	PD(SSB)	PD(DSB)
4.21000	0.00062	0.00195	0.00176
4.22000	0.00060	0.00189	0.00171
4.23000	0.00058	0.00183	0.00165
4.24000	0.00056	0.00177	0.00160
4.25000	0.00054	0.00171	0.00154
4.26000	0.00052	0.00166	0.00149
4.27000	0.00050	0.00161	0.00144
4.28000	0.00048	0.00155	0.00139
4.29000	0.00046	0.00150	0.00135
4.30000	0.00045	0.00145	0.00130
4.31000	0.00043	0.00140	0.00126
4.32000	0.00041	0.00136	0.00121
4.33000	0.00040	0.00131	0.00117
4.34000	0.00038	0.00127	0.00113
4.35000	0.00037	0.00122	0.00109
4.36000	0.00035	0.00118	0.00105
4.37000	0.00034	0.00114	0.00102
4.38000	0.00033	0.00110	0.00098
4.39000	0.00031	0.00106	0.00095
4.40000	0.00030	0.00103	0.00091
4.41000	0.00029	0.00099	0.00088
4.42000	0.00028	0.00095	0.00085
4.43000	0.00027	0.00092	0.00082
4.44000	0.00026	0.00089	0.00079
4.45000	0.00025	0.00085	0.00076
4.46000	0.00024	0.00082	0.00073
4.47000	0.00023	0.00079	0.00070
4.48000	0.00022	0.00076	0.00067
4.49000	0.00021	0.00073	0.00065
4.50000	0.00020	0.00071	0.00062
4.51000	0.00019	0.00068	0.00060
4.52000	0.00018	0.00065	0.00057
4.53000	0.00017	0.00063	0.00055
4.54000	0.00017	0.00060	0.00053
4.55000	0.00016	0.00058	0.00051
4.56000	0.00015	0.00055	0.00049
4.57000	0.00014	0.00053	0.00047
4.58000	0.00014	0.00051	0.00045
4.59000	0.00013	0.00049	0.00043
4.60000	0.00013	0.00047	0.00041
4.61000	0.00012	0.00045	0.00039
4.62000	0.00011	0.00043	0.00037
4.63000	0.00011	0.00041	0.00036
4.64000	0.00010	0.00039	0.00034
4.65000	0.00010	0.00037	0.00032
4.66000	0.00009	0.00036	0.00031
4.67000	0.00009	0.00034	0.00029
4.68000	0.00008	0.00032	0.00028
4.69000	0.00008	0.00031	0.00027
4.70000	0.00008	0.00029	0.00025
4.71000	0.00007	0.00028	0.00024
4.72000	0.00007	0.00026	0.00023
4.73000	0.00006	0.00025	0.00021
4.74000	0.00006	0.00023	0.00020
4.75000	0.00006	0.00022	0.00019
4.76000	0.00005	0.00021	0.00018
4.77000	0.00005	0.00020	0.00017
4.78000	0.00005	0.00018	0.00016
4.79000	0.00004	0.00017	0.00015
4.80000	0.00004	0.00016	0.00014

III. CONCLUSIONS

Analysis of a square wave modulated SSB signal shows that the presence of peaks in the waveform envelope constitutes a vulnerability to detection by an uncooperative receiver. This susceptibility to detection is present in various degrees under a wide range of bandwidths and signal to noise ratios.

The definition and magnitude of the peaks increase with increasing bandwidth of the transmitted signal, and this results in a greater degree of detectability of the waveform. Under practical conditions a signal to noise ratio of -10 db or less is required to obscure the peaks in the SSB signal to a point where detection is unlikely.

A detection technique combining a high threshold and an integration procedure is one method which demonstrates the susceptibility of the SSB signal to detection. SSB signals are significantly more vulnerable to this procedure than are comparable DSB signals. Detection by this technique is sensitive to bandwidth changes of the transmitted waveform.

In this discussion the detectability criterion used for comparison purposes is the envelope exceeding a threshold. Processing or displaying the received signal in some other manner will generally yield different detection probabilities for both DSB and SSB waveforms. Additional investigation of the detectability of spread spectrum signals is recommended.

A broad survey including comparisons of results obtained by threshold detectors, squaring receivers, and scanning spectrum analyzers would be particularly useful.

APPENDIX A

DERIVATION OF EXPRESSION FOR THE ENVELOPE E(t) OF A SQUARE-WAVE MODULATED SSB CARRIER

Figure 1 shows one method of generating a SSB signal. Let d(t) be a square wave of period T, so that

$$\omega_o = \frac{2\pi}{T} = \text{fundamental radian frequency}$$

Then, d(t) can be expressed as a fourier series,

$$d(t) = \frac{4}{\pi}\left(\cos \omega_o t - \frac{1}{3}\cos 3\omega_o t + \frac{1}{5}\cos 5\omega_o t - \ldots\right) \tag{21}$$

The AM-DSB/SC signal $v_a(t)$ is,

$$v_a(t) = d(t)\cos \ _c t$$

where, $f_c = \frac{\omega_c}{2\pi}$ is the carrier frequency in Hertz.

Substituting for d(t) from equation (21) and using a trigonometric identity gives,

$$v_a(t) = \frac{2}{\pi}\left[\cos(\omega_c + \omega_o)t - \frac{1}{3}\cos(\omega_c + 3\omega_o)t + \ldots\right] + \frac{2}{\pi}\left[\cos(\omega_c - \omega_o)t - \frac{1}{3}\cos(\omega_c - 3\omega_o)t + \ldots\right] \tag{22}$$

Each sideband can be readily identified in equation (22). The effect of the bandpass filter is to suppress one sideband,

which can be accomplished analytically in an ideal case by simply writing

$$v_c(t) = \frac{2}{\pi}\left[\cos(\omega_c + \omega_o)t - \frac{1}{3}\cos(\omega_c + 3\omega_o)t + \dots\right] \tag{1}$$

In this case, the lower sideband has been suppressed.

The envelope of the SSB signal $v_c(t)$ is of particular interest. Therefore, we express the last equation as,

$$v_c(t) = E(t)\cos\left[\omega_c t + \phi(t)\right] \tag{3}$$

where, $E(t)$ is the instantaneous envelope, and

$\phi(t)$ is the instantaneous phase angle.

By making use of a trigonometric identity, equation (1) can be written as,

$$\begin{aligned} v_c(t) = \frac{2}{\pi}\Big[& \cos\omega_c t\cos\omega_o t + \sin\omega_c t + \sin\omega_o t \\ & - \frac{1}{3}\cos\omega_c t\cos 3\omega_o t - \frac{1}{3}\sin\omega_c t\sin 3\omega_o t \\ & + \frac{1}{5}\cos\omega_c t\cos 5\omega_o t + \frac{1}{5}\sin\omega_c t\sin 5\omega_o t - \dots\Big] \end{aligned}$$

or,

$$\begin{aligned} v_c(t) = {} & \cos\omega_c t\left[\frac{2}{\pi}\left(\cos\omega_o t - \frac{1}{3}\cos 3\omega_o t + \frac{1}{5}\cos 5\omega_o t - \dots\right)\right] \\ & + \sin\omega_c t\left[\frac{2}{\pi}\left(\sin\omega_o t - \frac{1}{3}\sin 3\omega_o t + \frac{1}{5}\sin 5\omega_o t - \dots\right)\right] \end{aligned} \tag{24}$$

This equation is in the form of in phase and quadrature components, such as,

$$v_c(t) = x(t) \cos \omega_c t + y(t) \sin \omega_c t \qquad (25)$$

which transforms [Ref. 5] to,

$$v_c(t) = E(t) \cos(\omega_c t + \phi(t))$$

$$\text{with,} \quad E(t) = \left[x^2(t) + y^2(t) \right]^{1/2} \qquad (23)$$

$$\phi(t) = \tan^{-1} \frac{y(t)}{x(t)}$$

Applying equation (23) to equation (24) gives,

$$E(t) = \left\{ \left[\frac{2}{\pi} \left(\cos \omega_o t - \frac{1}{3} \cos 3\omega_o t + \cdots\cdots \right) \right]^2 + \left[\frac{2}{\pi} \left(\sin \omega_o t - \frac{1}{3} \sin 3\omega_o t + \ldots \right] ^2 \right\}^{1/2} \qquad (5)$$

which is equation (5) of this report.

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